4. (6+2+3+4 points) For Valentine's Day last year, Hannah brought her officemates chocolate. As the day progressed, the amount of chocolate remaining in the office decreased. After two hours, there were only 5 pounds of chocolate remaining, and after seven hours, there was only 1 pound left.

(a) Assuming that the amount of chocolate in the office decreased linearly, write an equation for the amount of chocolate, c in pounds, left after t hours.

$$c(2) = 5 \text{ and } c(7) = 1.$$

$$\text{slope} = \frac{\text{change in chocolate}}{\text{change in time}} = \frac{5-1}{2-7} = \frac{-4}{5}.$$

Thus, we have that $c(t) = \frac{-4}{5}t + b$.

To solve for b, substitute into the equation one of the given points: $\frac{-4}{5}(2) + b = 5$.

This gives $b = \frac{33}{5}$ and $c(t) = \frac{-4}{5}t + \frac{33}{5}$.

(b) How much chocolate did Hannah bring to the office?

The amount of chocolate that Hannah brought to the office is given by $c(0)$. Plugging in zero for t into the equation from (a) gives $\frac{33}{5}$ or 6.6 pounds of chocolate.

(c) What is the practical interpretation of the slope of your linear function in the context of this problem?

Every five hours, Hannah's officemates eat four pounds of chocolate.

(d) Now, assume instead that the amount of chocolate left at time t was represented by the exponential function $C(t) = ab^t$, find a and b, and express your answer as a function. [Do not assume that this function indicates the same beginning amount of chocolate as in part (a). Use the data given in the original statement of the problem to determine a and b.]

$$c(2) = 5 \text{ and } c(7) = 1$$

$$\Rightarrow ab^2 = 5 \text{ and } ab^7 = 1$$

$$\Rightarrow \frac{ab^2}{ab^7} = \frac{5}{1}$$

$$\Rightarrow b^{-5} = 5$$

$$\Rightarrow b = \frac{1}{5}^{\frac{1}{5}} \approx .725$$

$$\Rightarrow a(.725)^2 = 5$$

$$\Rightarrow a = 9.5$$

$$\Rightarrow c(t) = 9.5(.725)^t.$$