

5. (8 points) After a dementor attack, Harry, the wizard, eats chocolate in order to feel better. When a wizard eats chocolate, the chocolate enters their bloodstream instantaneously and the body metabolizes and eliminates it from the bloodstream at the rate of 20% per hour.

(a) If Harry ate $1/2$ pound of chocolate, write a formula for the amount of chocolate, Q (in pounds), remaining in his bloodstream t hours after he ate the chocolate.

Harry ate a $1/2$ pound of chocolate, so $Q_0 = 1/2$. We're told that his body metabolizes and eliminates it from the bloodstream at the rate of 20% per hour. Since the function is changing at a constant *percent*, we have an exponential function with the growth factor of $(1 - .20) = .80$. Thus,

$$Q(t) = \frac{1}{2}(0.8)^t.$$

(b) If Harry's chocolate level in his bloodstream becomes lower than 0.2 pounds, he will go into shock. What is the maximum amount of time, t , that he can wait before eating more chocolate? Show your work.

The maximum value of time, t , that Harry can wait is the value of t that satisfies the equation $0.2 = \frac{1}{2}(0.8)^t$.

$$\Rightarrow 0.4 = (0.8)^t$$

$$\Rightarrow \ln(0.4) = t * \ln(0.8)$$

$$\Rightarrow t = \frac{\ln(0.4)}{\ln(0.8)}$$

So, $t \approx 4.1$ hours.

6. (5 points) Let $f(x) = \sin(3x^2)$. Use the **definition** of the derivative to express $f'(2)$ as a limit. You do **not** need to simplify your expression or try to approximate $f'(2)$.

$$\begin{aligned} f'(2) &= \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sin(3(2+h)^2) - \sin(12)}{h} \end{aligned}$$