8. (3 points each) Sarah decided to run a marathon. However, she started off way too fast and so her speed decreased throughout the race. Below is a table showing how many miles she had run at time t minutes since the beginning of the race.

time (min)	30	60	90	120	150	180	210	240
distance (miles)	5	9	12.5	15.5	18.5	21	23.25	25.2

Let s be the function such that $s(t)$ is Sarah's distance from the starting line t minutes after the race began.

(a) What is the practical interpretation of $s'(120)$ in the context of this problem?

The expression $s'(120)$ gives Sarah's speed, in miles/min, two hours into the race.

(b) Estimate $s'(120)$.

We must estimate the instantaneous rate of change at $t = 120$. You can estimate this value "from the left", "from the right", or "both the left and the right and take their average". In this case, all will give you the same answer. Here, we estimate from the right:

$$s'(120) \approx \frac{18.5 - 15.5}{150 - 120} = 0.1\text{miles/min}.$$

(c) What is the practical interpretation of $s^{-1}(14)$ in the context of this problem?

The expression $s^{-1}(14)$ gives the length of time that Sarah has been running when she is 14 miles into the race.

(d) Estimate $s^{-1}(14)$.

Notice that at $t = 90$, Sarah is at the 12.5 mile mark and at $t = 120$, Sarah is at the 15.5 mile mark. So, sometime between $t = 90$ and $t = 120$ Sarah passes the 14 mile mark. Since the average speed between mile 12.5 and 15.5 is approximately 1/10 miles/min, to go the 1.5 miles would take approximately 15 minutes. Thus, $s^{-1}(14)$ is approximately equal to $90 + 15 = 105$ minutes.

(e) What does the derivative of $s^{-1}(P)$ at $P = 14$ represent in the context of this problem?

Once Sarah has run 14 miles, $(s^{-1})'(14)$ gives the approximate amount of time it will take her to run the next mile (assuming that her pace stays the same for the next mile). The units of $(s^{-1})'(14)$ are in minutes/mile.

(f) Estimate the derivative of $s^{-1}(P)$ at $P = 14$.

$$(s^{-1})'(14) \approx \tfrac{120-105}{15.5-14} = 10 \text{ minutes/mile}$$