

8. (15 points) The functions $r = f(t)$ and $V = g(r)$ give the radius and the volume of a commercial hot air balloon that is being inflated for testing. The variables t and r are measured in minutes and feet respectively, while the volume V is measured in cubic feet. The inflation begins at $t = 0$.

In each case, translate the words or phrases given below into the precise mathematical expression that represents them. This mathematical expression will consist *only* of numbers, variables and symbols defined in this problem, and other mathematical symbols related to function notation and operations. For example, in problem 1(b) the notation “ $p'(6)$ ” is a mathematical expression. There we gave an expression and you were asked to interpret. Here we are giving the interpretation and you are to supply the mathematical expression.

You may assume V and r are strictly increasing, differentiable functions.

- (a) The average rate of change in the volume of the balloon when the radius expands from 10 to 12 feet:

Mathematical Expression $(g(12) - g(10))/2$

- (b) The volume of the balloon t minutes after inflation began:

Mathematical Expression $g(f(t))$

- (c) The volume of the balloon if the radius was twice as big:

Mathematical Expression $g(2r)$

- (d) The time elapsed when the radius of the balloon is 30 feet:

Mathematical Expression $f^{-1}(30)$

- (e) The time elapsed when the volume of the balloon is 10,000 cubic feet:

Mathematical Expression $f^{-1}(g^{-1}(10,000))$