1. According to a survey by the U-M Transportation Research Institute, gasoline prices are projected to reach $5.00 a gallon by the year 2020.

   (a) (5 points) Assuming that the average gas price in 2007 is $2.00 per gallon (yes, we know that is wishful thinking), find an exponential function, P, that models the average gas price t years after 2007. Show either an "exact" answer or at least 4 decimal places in your answer.

   Since the function we are looking for is exponential, we are looking for a function of the form $P = ab^t$, where a and b are constants, passing through the points $(0, 2)$ and $(13, 5)$.
   Using the point $(0, 2)$ we have $0 = ab^0 \Rightarrow a = 2$. Now, using the point $(13, 5)$, combined with what we just showed we have $5 = 2b^{13} \Rightarrow b = (\frac{5}{2})^{\frac{1}{13}}$.
   Thus the function we are looking for is $P = 2(\frac{5}{2})^{\frac{t}{13}}$ or approximating to four decimal places $P = 1.0730^t$.

   (b) (2 points) What is the *annual* percent change in the average gas price according to this model? (Show to at least one decimal place.)

   To find the annual percent change in gas prices we note that the value b we calculated in (a) was approximately 1.0730. Thus the annual percent change in gas prices is (to one decimal place) 7.3%.

   (c) (2 points) What is the yearly *continuous* percent rate of change for this model? (Show to two decimal places.)
   To find the continuous percent rate of change for the model we have to express $P(t)$ in the form $P(t) = ae^{kt}$, for constants a and k.
   $P(t) = 2(\frac{5}{2}^{\frac{1}{13}})^t = ae^{kt} \Rightarrow a = 2$ and $e^k = (\frac{5}{2})^{\frac{1}{13}}$. Solving this for k we get $k = \frac{1}{13}ln(\frac{5}{2}) \approx 0.0705$. Thus, the continuous growth rate is 7.05%.

   (d) (5 points) If, instead, gasoline prices grow linearly between 2007 and 2020, find a linear function, L, to model the price t years after 2007.

   Since the function we are looking for is linear, we are looking for a function of the form $P = mt + d$, where m and d are constants, passing through the points $(0, 2)$ and $(13, 5)$.
   The constant m is the slope, which we can calculate $m = \frac{5-2}{13-0} = \frac{3}{13}$. Since we have the point $(0, 2)$, we have $L = \frac{3}{13}t + 2$.

   (e) (2 points) The survey indicates that prices may be $4.00 per gallon eight years from now. Which of the two models best predicts this projection?

   The exponential model predicts an average price of approximately $3.51 per gallon, and the linear model predicts an average price of approximately $3.85 per gallon. Thus the linear model is the better model.