6. (16 points) State whether each of the following statements are TRUE or FALSE. For each statement, give an explanation. If the statement is false, give an example that shows a contradiction to the statement. If the statement is true, show why it is true. Examples may be formulas or graphs. Explain your reasoning.

   (a) If $f'(x)$ is increasing, then $f(x)$ is also increasing.

   FALSE
   Consider the function $f(x) = x^2$. We know that $f''(x) > 0$ for all x (so that $f'(x)$ is increasing), but $f'(x) = 2x$ is less than 0 for $x < 0$ so that f is decreasing for $x < 0$.

   (b) If $f(x) \neq g(x)$ for all x, then $f'(x) \neq g'(x)$.

   FALSE
   Consider $f(x) = x + 1$ and $g(x) = x + 2$. Then $f'(x) = g'(x) = 1$ even though $f(x) \neq g(x)$.

   (c) There is a function which is continuous on [1,5] but not differentiable at $x = 3$.

   TRUE
   The function $f(x) = |x - 3|$ is one such function.

   (d) If a function is increasing on an interval, then it is concave up on that interval.

   FALSE
   The function $f(x) = \ln x$ is a counterexample.