

9. (8 points) Cosmologists, through a technique best described as hocus pocus, measure a quantity $T(t)$, the temperature of the universe in degrees Kelvin (K), where t is in gigayears (Gyr) after the Big Bang. Suppose that, currently, $t = 13.6$, $T(13.6) = 2.4$, and $T'(13.6) = -12$.

[Note: A gigayear is 1 billion years, and the Kelvin temperature scale is an absolute temperature scale where the lowest possible temperature is defined as being zero Kelvin.]

- (a) For each of the following statements, state whether you agree or disagree with the conclusion and justify your reasoning.
- (i) In the next billion years, the temperature of the universe will drop by approximately 12 degrees Kelvin.

It is not possible for the temperature of the universe to drop by 12 Kelvin in the next billion years because the current temperature is 2.4 Kelvin and the Kelvin temperature scale is absolute (the lowest possible temperature is 0 Kelvin).

- (ii) In the next year, the temperature of the universe will drop by approximately $\frac{12}{1,000,000,000}$ degrees Kelvin.

It is reasonable to say that the temperature of the universe will drop by approximately $\frac{12}{1,000,000,000}$ Kelvin in the next year since $T'(13.6) = -12$ so as time increases by one billionth of a Gyr from the current time ($t = 13.6$ Gyr) the temperature should decrease by twelve billionths Kelvin.

- (b) Assume $T(t)$ is decreasing and does not change concavity on the domain $[13.6, \infty)$. Do you expect $T(t)$ to be concave up or concave down on the domain $[13.6, \infty)$? Justify your answer using physical reasoning.

The function $T(t)$ should be concave up since as the universe expands, the temperature of the universe decreases and $T(t)$ should asymptotically approach zero.