

3. You are driving to Detroit to see a concert at the Majestic Theater. You leave Ann Arbor at 6:00 pm. Let  $D(t)$  be your distance from Detroit  $t$  minutes after 6:00.

- (a) (3 points) What is the sign (positive or negative) of  $D'(t)$ , assuming you never turn around on your way to Detroit? Explain.

$D'(t)$  must be negative, since the function  $D(t)$  is a decreasing function: the distance to Detroit decreases as you drive towards the city.

As you approach the city, you notice signs indicating the distance remaining. To pass the time, your friend, riding in the passenger seat, makes the following table:

$t$ , minutes after 6:00	0	3	6	10	15	18	20	22	25	30
$D(t)$ , miles from Detroit	50	47	43	38	35	35	32	30	27	20

- (b) (4 points) Use the table to estimate  $D'(6)$ . Include units.

Using the difference quotient from the left, we have

$$D'(6) \approx \frac{D(6) - D(3)}{6 - 3} = \frac{43 - 47}{6 - 3} = -\frac{4}{3} \approx -1.33.$$

From the right, we have

$$D'(6) \approx \frac{D(10) - D(6)}{10 - 6} = \frac{38 - 43}{10 - 6} = -\frac{5}{4} = -1.25.$$

Averaging the two, we have  $D'(6) \approx -\frac{31}{24} \approx -1.29$ . In any case, the units are in miles per minute.

- (c) (3 points) Based on your answer to (??), approximately what did your speedometer read at 6:06? (Your car's speedometer gives speed in miles/hour.)

Taking the absolute value of the answer to (??), and converting units to miles per hour by multiplying by 60, we see that the speedometer said approximately  $60 \cdot \frac{4}{3} = 80$  miles per hour. (Or  $60 \cdot \frac{5}{4} = 75$  mi/hr, or  $60 \cdot 1.29 = 77.4$  mi/hr, depending on which of the three methods used to solve (??).)

- (d) (4 points) Could  $D(t)$  be linear ...

- for  $20 \leq t \leq 30$ ? Briefly explain.

No. If it were, its graph would have slope  $-1 = \frac{30-32}{22-20}$  for  $20 \leq t \leq 22$ , but slope  $-\frac{7}{5} = \frac{20-27}{30-25}$  for  $25 \leq t \leq 30$ .

- for  $20 \leq t \leq 25$ ? Briefly explain.

This is possible, because the difference quotients  $\frac{30-32}{22-20}$ ,  $\frac{37-30}{25-22}$  are both equal to  $-1$ .