4. (6 points) A certain state has been setting the date for its primary election using a function \( P(x) \), where \( x \) is the number of years since 1992 and \( P(x) \) is the number of days from the beginning of the year when the primary was held. (Count January 1 as one day from the beginning.) The pattern of elections is given in the table:

\[
\begin{array}{c|c|c|c|c|c}
 x & 0 & 4 & 8 & 12 & 16 \\
 P(x) & 96 & 48 & 24 & 12 & 6 \\
\end{array}
\]

Assuming that \( P \) is either linear or exponential, write a formula for \( P(x) \) which accurately reflects the data in the table. If this trend continues, when will the primary be held in 2012? Show your work.

First, \( P \) cannot be linear, since \( \frac{P(4) - P(0)}{4 - 0} = \frac{48 - 96}{4} = -12 \), but \( \frac{P(8) - P(4)}{8 - 4} = \frac{24 - 48}{4} = -6 \). Assuming \( P \) is exponential, then, write \( P(x) = C \cdot b^x \). Since \( P(0) = C \), we have \( C = 96 \). Since \( \frac{P(4)}{P(0)} = \frac{C \cdot b^4}{C \cdot b^0} = b^4 \), we have \( b^4 = \frac{48}{96} = \frac{1}{2} \), so \( b = \sqrt[4]{\frac{1}{2}} \approx 0.84 \). (Note: taking the negative 4th root \( b = -0.84 \) doesn’t make sense in the context of the problem.) Thus

\[ P(x) = 96 \left( \sqrt[4]{\frac{1}{2}} \right)^x \approx 96(0.84)^x, \]

and when \( x = 20 \) (i.e, the year 2012) \( P(x) = 3 \). The primary will take place on January 3rd in 2012.

5. (8 points) On the axes below, carefully sketch the graph of a continuous function \( f(x) \) with the following properties:

- \( f \) is an even function (that is, \( f(-x) = f(x) \)).
- \( f(0) = 1 \).
- \( f'(x) = -2 \) on \((-2, 0)\).
- \( f'(x) < 0 \) for \( x > 2 \).
- \( f''(x) > 0 \) for \( x < -2 \).
- \( \lim_{x \to \infty} f(x) = -1 \).

Your graph should be as accurate as possible. (You won’t be graded on your draftsmanship, though!)