4. (6 points) A certain state has been setting the date for its primary election using a function $P(x)$, where x is the number of years since 1992 and $P(x)$ is the number of days from the beginning of the year when the primary was held. (Count January 1 as one day from the beginning.) The pattern of elections is given in the table:

x	0	4	8	12	16
$P(x)$	96	48	24	12	6

Assuming that P is either linear or exponential, write a formula for $P(x)$ which accurately reflects the data in the table. If this trend continues, when will the primary be held in 2012? Show your work.

First, P cannot be linear, since $\frac{P(4)-P(0)}{4-0} = \frac{48-96}{4-0} = -12$, but $\frac{P(8)-P(4)}{8-4} = \frac{24-48}{8-4} = -6$. Assuming P is exponential, then, write $P(x) = C \cdot b^x$. Since $P(0) = C$, we have $C = 96$. Since

$$\frac{P(4)}{P(0)} = \frac{C \cdot b^4}{C \cdot b^0} = b^4,$$

we have $b^4 = 48/96 = 1/2$, so $b = \sqrt[4]{1/2} \approx 0.84$. (Note: taking the negative 4th root $b = -0.84$ doesn't make sense in the context of the problem.) Thus

$$P(x) = 96(\sqrt[4]{1/2})^x \approx 96(0.84)^x,$$

and when $x = 20$ (i.e, the year 2012) $P(x) = 3$. The primary will take place on January 3^{rd} in 2012.

5. (8 points) On the axes below, carefully sketch the graph of a continuous function $f(x)$ with the following properties:

- f is an even function (that is, $f(-x) = f(x)$).
- $f(0) = 1$.
- $f'(x) = -2$ on $(-2, 0)$.
- $f'(x) < 0$ for $x > 2$.
- $f''(x) > 0$ for $x < -2$.
- $\lim_{x\to\infty} f(x) = -1$.

Your graph should be as accurate as possible. (You won't be graded on your draftsmanship, though!)