7. You've arrived at the Majestic Theater, excited to see your favorite immigrant gypsy punk band play. As soon as the show starts, some people from the audience try to climb onto the stage. Once audience members are on the stage, they refuse to leave the stage. So, the theater has a security team in charge of pushing these people off the stage. Let

- $P(t)$ be the total number of people on the stage;
- $B(t)$ be the number of band members on the stage;
- $S(t)$ be the number of security guards on the stage;
- $A(t)$ be the number of audience members on the stage;
- $c(t)$ be the rate at which audience members are climbing onto the stage;
- $d(t)$ be the rate at which audience members are being pushed off the stage by security;

all at time t, where t the is number of minutes after the show begins. Since there are so many people at the show, you can assume these are all differentiable functions of t.

(a) (2 points) What does $P(t) - (B(t) + S(t))$ represent? (Assume everyone in the theater is either in the band, in the audience, or a security guard.)

$P(t) - (B(t) + S(t)) = A(t)$, the number of audience members on stage at time t.

(b) (2 points) Write an equation for $A'(t)$ in terms of the appropriate functions from the list above.

$A'(t) = c(t) - d(t)$.

(c) (2 points) In the context of this problem, is it possible that $c(t) < 0$?

No. (It's hard to make sense of a negative rate of people climbing onto the stage, especially since we're told that no one climbs off.)

(d) (2 points) After one hour, the lead singer tells the guards to stop pushing people off the stage. What does this mean about $d(t)$, for $t > 60$?

This means $d(t) = 0$ for all $t > 60$.

(e) (2 points) The show lasts a total of two hours. On the interval $[0, 60]$, is $A(t)$ increasing, decreasing, neither, or is there insufficient information? Explain.

There is not enough information: the sign of $A'(t) = c(t) - d(t)$ could be positive or negative on the interval $[0, 60]$.

(f) (2 points) On the interval $[60, 120]$, is $A(t)$ increasing, decreasing, neither, or is there insufficient information? Explain.

$A(t)$ is non-decreasing on $[60, 120]$. We know $A'(t) = c(t) - d(t) = c(t) \geq 0$ for t in $[60, 120]$, because $d(t) = 0$ on this interval, and $c(t) \geq 0$ always. [Answers here could be "increasing," "non-decreasing," or "insufficient information." The explanation was important in all cases.]