

8. Census figures for the US population (in millions) are listed in the table below. Let  $f$  be the function such that  $P = f(t)$  is the population (in millions) at year  $t$ .

Year	1950	1960	1970	1980	1990
Pop.	150.7	179.0	205.0	226.5	248.7

Assume that  $f$  is increasing, so  $f$  is invertible.

- (a) (3 points) What is the meaning of  $f^{-1}(200)$ ?

This is the year when the US population was 200 million.

- (b) (3 points) What does the derivative of  $f^{-1}(P)$  at  $P = 200$  represent? What are its units?

The units of  $(f^{-1})'(200)$  are years per millions of people. This number represents the approximate time (in years) for the population of the U.S. to grow from 200 million to 201 million people.

- (c) (3 points) Estimate  $f^{-1}(200)$ .

$f^{-1}(200)$  must be between 1960 and 1970. If the increase is linear, the population during that period is increasing by 2.6 million people per year. Reasonable estimates are 1968 or 1967, because the rate of growth appears to be slowing down. [Again, answers may vary, but work and reasons were important.]

- (d) (3 points) Estimate the derivative of  $f^{-1}(P)$  at  $P = 200$ .

Using the estimate from (??) and the difference quotient from the right, we have

$$(f^{-1})'(200) \approx \frac{f^{-1}(205.0) - f^{-1}(200)}{205.0 - 200} = \frac{1970 - 1968}{205 - 200} = \frac{2}{5} = 0.4.$$