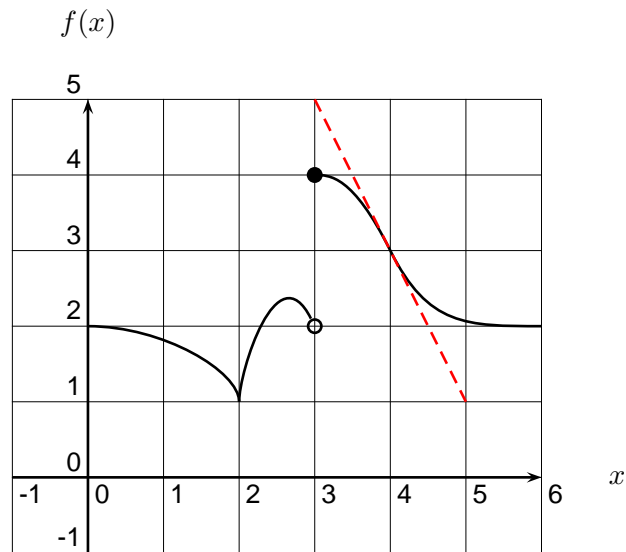9. A function f is defined on the interval $[0, 6]$. The graph of $y = f(x)$ is shown below.

(a) (2 points) On which intervals does it appear that f is continuous?

Judging from the graph, f appears to be continuous on $[0, 3)$ and on $(3, 6]$. It is not continuous at $x = 3$, because the value jumps there.

(b) (3 points) On which intervals does it appear that f is differentiable?

It appears that f is differentiable on $(0, 2)$, $(2, 3)$, and $(3, 6)$. It cannot be differentiable at $x = 3$, because it is not continuous there. At $x = 2$, there is a sharp point, suggesting that f is not differentiable there, either.

(c) (3 points) Does $\lim_{x \to 3} f(x)$ exist? If so, estimate it; if not, explain why.

$\lim_{x \to 3} f(x)$ does not exist, because $f(x)$ approaches 2 as x approaches 3 from the left, but $f(x)$ approaches 4 as x approaches 3 from the right. Since these are different values, the limit cannot exist.

(d) (4 points) Estimate $f'(4)$ and find an equation of the tangent line to the graph of f at $x = 4$.

Drawing the tangent line, we it has slope -2, so $f'(4) = -2$. Using the point $(4, 3)$, we can write the equation of the tangent line as

$$(y - 3) = -2(x - 4),$$

or equivalently,

$$y = -2x + 11.$$