

1. Air pressure,  $P$ , decreases exponentially with the height,  $h$ , in meters above sea level. The unit of air pressure is called an *atmosphere*; at sea level, the air pressure is 1 atm.

- (a) (5 points) On top of Mount McKinley, at a height of 6198 meters above sea level, the air pressure is approximately 0.48 atm. Use this to determine the air pressure 12 km above sea level, the maximum cruising altitude of a commercial jet.

Since we know  $P$  is a decreasing exponential function of  $h$ ,

$$P(h) = P_0 e^{kh}$$

for some constants  $k$  and  $P_0$ . Plugging in  $h = 0$ , we find that  $P_0 = 1$ . Plugging in  $h = 6198$ , we obtain the equation

$$0.48 = e^{6198k}.$$

Taking the natural logarithm of both sides and dividing by 6198, we find  $k \approx -1.184 \cdot 10^{-4}$ . Using this value of  $k$ , we find that

$$P(12000) = e^{-12000k} \approx 0.241 \text{ atm.}$$

- (b) (4 points) Determine  $P^{-1}(0.7)$ . Include units!

We know that  $P^{-1}(0.7)$  gives us the height above sea level at which the air pressure is 0.7 atm. Thus, we want to solve the equation

$$0.7 = e^{-1.184 \cdot 10^{-4}h}$$

for  $h$ . Thus,

$$\ln 0.7 = -1.184 \cdot 10^{-4}h,$$

$$\text{so } h = \frac{\ln 0.7}{-1.184 \cdot 10^{-4}} \approx 3012.46 \text{ m (or } \approx 3.012 \text{ km).}$$