

2. A new company produces and sells socks. By far their most successful item is the business sock (that's why they call them business socks), and the company hires a young consultant to assess the impact of advertising this popular product. Let S denote the yearly sales revenue, in thousands of dollars, and a denote the annual advertising expenditure, also in thousands of dollars. The company assumes that sales revenue will depend on advertising, so we write $S = f(a)$.

- (a) (2 points) What does the company hope is true about the sign of f' ? Explain.

It would be reasonable for the company to expect $f' > 0$, since this simply means that they sell more socks as they spend more on advertising.

- (b) (2 points) The consultant suggests that $\lim_{a \rightarrow \infty} f'(a) = 0$. Is this reasonable? Why or why not?

This is a reasonable assumption, since eventually all the people potentially interested in purchasing business socks will have seen an advertisement; putting more money in at that point doesn't change the situation much. In the language of economics, marginal revenue is decreasing. [Note: other answers are accepted—with reasonable justification.]

- (c) The consultant makes the following statements. Interpret her observations in practical terms. Do not use the word "rate"!

- i. (3 points) $f(0) = 3$

Even if the company doesn't spend any money on advertising, they will sell \$3000 worth of business socks per year.

- ii. (3 points) $f'(0) = 4$

If the company spends \$1000 on advertising, their sales revenue should increase by approximately \$4000.

- iii. (3 points) $f^{-1}(6.6) = 1$

If the company spends \$1000 on advertising, the sales revenue will be \$6600.

- iv. (3 points) $(f^{-1})'(6.6) = 0.31$

When the company's annual sales revenue is \$6600, increasing the sales revenue to \$7600 will require increasing spending on advertising by approximately \$310.