8. A continuous function f, defined for all x, is always decreasing and concave up. Suppose $f(6) = -6$ and $f'(6) = -1.5$.

   (a) (2 points) How many zeros does f have? Justify your answer.

   The function f is always decreasing, so it has at *most* one zero.

   On the other hand, since it is concave up, f always lies above the line $y = -1.5\,x + 3$ (the line tangent to f at the point $(6, -6)$). In particular, f takes positive values. It also takes negative values (e.g. at $x = 6$) and is continuous, and so must cross the x-axis somewhere. Thus, f has at *least* one zero, and combining this with the first statement we conclude that f has precisely one zero.

   (b) (2 points) Can $f'(2) = -1$? Justify your answer.

   Since f is concave up, f' must be increasing. Since $f'(6) = -1.5$, we must have $f'(2) \leq -1.5$.

   Therefore, the answer is NO.

   (c) (4 points) Circle all intervals below in which f has at least one zero. Justify your choices with a picture and a short description.

   i. $(-\infty, -6)$

   ii. $[-6, -2)$

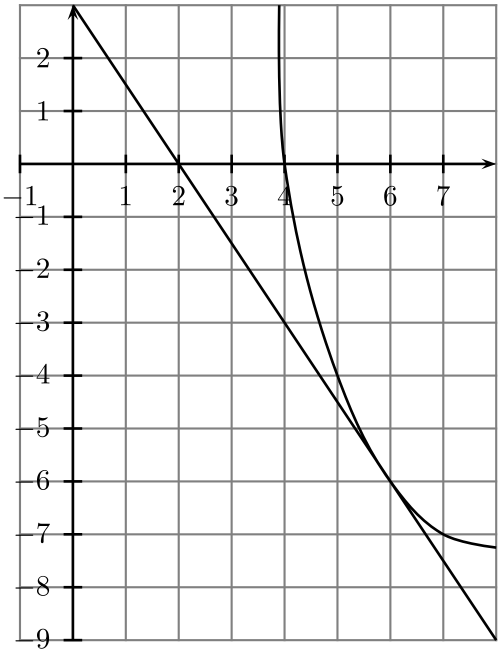
   iii. $[-2, -1)$

   iv. $[-1, 1)$

   v. $[1, 2)$

   vi. $\boxed{[2, 6)}$

   vii. $[6, \infty)$



From the reasoning given for part (a), we know f lies above the line tangent to f at $x = 6$. Thus, the smallest x at which f could have a zero is at 2. If f decreases quickly enough, it could have a zero arbitrarily close to (but to the left of) $x = 6$.