8. A continuous function $f$, defined for all $x$, is always decreasing and concave up. Suppose $f(6) = -6$ and $f'(6) = -1.5$.

(a) (2 points) How many zeros does $f$ have? Justify your answer.

The function $f$ is always decreasing, so it has at most one zero.

On the other hand, since it is concave up, $f$ always lies above the line $y = -1.5x + 3$ (the line tangent to $f$ at the point $(6, -6)$). In particular, $f$ takes positive values. It also takes negative values (e.g. at $x = 6$) and is continuous, and so must cross the $x$-axis somewhere. Thus, $f$ has at least one zero, and combining this with the first statement we conclude that $f$ has precisely one zero.

(b) (2 points) Can $f'(2) = -1$? Justify your answer.

Since $f$ is concave up, $f'$ must be increasing. Since $f'(6) = -1.5$, we must have $f'(2) \leq -1.5$.

Therefore, the answer is NO.

(c) (4 points) Circle all intervals below in which $f$ has at least one zero. Justify your choices with a picture and a short description.

i. $(-\infty, -6)$
ii. $[-6, -2)$
iii. $[-2, -1)$
iv. $[-1, 1)$
v. $[1, 2)$
vi. $[2, 6)$
vii. $[6, \infty)$

From the reasoning given for part (a), we know $f$ lies above the line tangent to $f$ at $x = 6$. Thus, the smallest $x$ at which $f$ could have a zero is at 2. If $f$ decreases quickly enough, it could have a zero arbitrarily close to (but to the left of) $x = 6$. 
