10. [8 points]
According to US Census Data, the population of the city of Detroit has been declining since 1950. Suppose that $P = f(t)$ is the population of the city of Detroit (in millions of people) t years after 1950. The table below gives some values of $P = f(t)$.

t	0	10	20	30	40	50
P	1.8496	1.6701	1.5115	1.2033	1.0280	0.9513

a. [4 points]
Use the table to estimate the derivative of $f^{-1}(P)$ at $P = 1.61$. Be sure to include units with your answer.

Solution: The points $(10, 1.6701)$ and $(20, 1.5115)$ are the closest to $P = 1.61$, so we will use these for our estimate. Slopes for f^{-1} are of the form $\frac{\Delta t}{\Delta P}$ (since P is the input of f^{-1} and t is the output). Between our two points, we have

$$\frac{\Delta t}{\Delta P} = \frac{20 - 10}{1.5115 - 1.6701} \approx -63.05,$$

so the derivative of $f^{-1}(P)$ at $P = 1.61$ is approximately -63.05 years per million people.

b. [4 points]
Suppose Detroit's population decays exponentially starting in 1990. In what year will Detroit have a population of 650,000 people?

Solution: We have two points in the period starting in 1990, namely $(40, 1.0280)$ and $(50, 0.9513)$. We will use these to find the exponential decay rate. In ten years, the population multiplies by $\frac{0.9513}{1.0280}$, so in one year, the population multiplies by $\left(\frac{0.9513}{1.0280}\right)^{\frac{1}{10}}$. If we let N be the number of years since 1990, then a formula for P after 1990 is given by

$$P = 1.0280\left(\left(\frac{0.9513}{1.0280}\right)^{\frac{1}{10}}\right)^N \approx 1.0280(0.992276)^N.$$

Set $P = 0.65$ and solve for N using logs.

$$0.65 = 1.0280\left(\left(\frac{0.9513}{1.0280}\right)^{\frac{1}{10}}\right)^N.$$

$$\ln\left(\frac{0.65}{1.0280}\right) = N \ln\left(\left(\frac{0.9513}{1.0280}\right)^{\frac{1}{10}}\right).$$

$$\frac{\ln\left(\frac{0.65}{1.0280}\right)}{\ln\left(\left(\frac{0.9513}{1.0280}\right)^{\frac{1}{10}}\right)} = N \approx 59.11.$$

Thus, if the population is decaying exponentially, there will be 650,000 people in Detroit in the year 2049, or 59.11 years after 1990.