

4. [10 points] Let $W = f(t)$ be the amount of water, in gallons, in a bathtub at time t , in minutes. Suppose that Anna turns on the water in the bathtub at time $t = 0$. After exactly 6 minutes, Anna turns off the water and proceeds to bathe her new puppy, Asta. Asta loves the water and doesn't splash or fuss, so Anna has the opportunity to give her a good shampoo. After a 10 minute bath, Anna pulls the plug, and the bathtub takes exactly 4.5 minutes to empty. Because Asta sheds, the water drains quickly at first and then the draining slows down. Anna keeps clearing the hair to assist the draining.

- a. [3 points] For which values of t between 0 and 25 is the quantity $\frac{dW}{dt}$...

positive?

Solution: $\frac{dW}{dt}$ is positive for $0 < t < 6$

zero?

Solution: $\frac{dW}{dt}$ is zero for $6 < t < 16$ and for $20.5 < t < 25$

negative?

Solution: $\frac{dW}{dt}$ is negative for $16 < t < 20.5$

- b. [2 points] Suppose that the line tangent to the graph of $W = f(t)$ at $t = 18$ passes through the points $(17, 17.5)$ and $(18.5, 4)$. Find $f'(18)$ and include units with your answer.

Solution: $f'(18) = \frac{4 - 17.5}{18.5 - 17} = -9$ gallons per minute

- c. [2 points] Find $f(18)$ and include units with your answer.

Solution: Since the slope at $t = 18$ is -9 and the point $(18, f(18))$ is on the tangent line, we can subtract 9 from $f(17)$. Thus, $f(18) = 8.5$ gallons.

- d. [3 points] Use your answers from (b) and (c) to estimate $f(20)$. Explain, in practical terms, why this estimate is or is not reasonable.

Solution: Using the information from (b) and (c), the estimate for $f(20) \approx 8.5 + 2(-9) = -9.5$ gallons. Clearly this estimate is unreasonable, since there cannot be a negative number of gallons in the bathtub. Alternate explanation: we know $f(20)$ (the number of gallons in the tub after 20 minutes) must be positive, since the bathtub doesn't finish emptying until $t = 20.5$.