1. [12 points] Cyanide is used in solution to isolate elemental gold in gold mines. This unfortunately may result the groundwater near mines being contaminated with cyanide, which then must be removed. Suppose that at a certain mine site cyanide is removed from the groundwater starting in 2005. The concentration, c (in ppm), of cyanide found in the groundwater at the site t years after the year 2005 is given in the following table.

t (years)	0	1	2
c (ppm)	25.0	21.8	18.9

(Values for c are rounded.)

a. [4 points] Find the equation of an exponential model for $c(t)$. Show your work.

Solution: Let $c(t) = c_0\, a^t$. Then $c(0) = c_0 = 25$ and $c(1) = 25\,a = 21.8$, so $a = 21.8/25 = 0.87$. Thus $c(t) = 25\,(0.87)^t$. Note that $c(2) = 25\,(0.87)^2 = 18.9$, the value given in the table. Alternately, let $c(t) = c_0\, e^{kt} = 25\, e^{kt}$. Then $c(1) = 25\, e^k = 21.8$, so that $k = \ln(21.8/25) = \ln(0.87) = -0.14$. Thus we may also write $c(t) = 25\, e^{-0.14t}$. As before, this gives $c(2) = 18.9$.

b. [4 points] Using your equation from (a), how many years will it take for the concentration of cyanide to be reduced to 10 ppm? Show all of your work.

Solution: We want $c(t) = 25\,(0.87)^t = 10$, so that $t = \ln(10/25)/\ln(.87) = 6.6$ years. Alternately, with $c(t) = 25\, e^{-0.14t} = 10$, we have $t = \ln(10/25)/(-0.14) = 6.6$ years as well.

c. [2 points] The cyanide removal process involves pumping groundwater through a filtering system. Suppose that the speed of this pumping process is doubled from the pumping speed which produced the data given above. Call the resulting concentration function $c_1(t)$. Use your expression for $c(t)$ from (a) to write an equation for $c_1(t)$.

Solution: If we double the pumping rate, then we expect our new $c(t)$ to be $c_1(t) = c(2t) = 25\,(0.87)^{2t}$ $(= 25(0.76)^t = 25e^{-0.27t})$.

d. [2 points] Now instead suppose that the groundwater cleaning started 3 years earlier. Call the resulting concentration function $c_2(t)$. Use your expression for $c(t)$ from (a) to write an equation for $c_2(t)$.

Solution: Starting 3 years earlier, we have $c_2(t) = c(t+3) = 25\,(0.87)^{t+3}$ $(= 16.5\,(0.87)^t)$.