**2**. [12 points] Consider the following table giving values, rounded to three decimal places, of a function $f(x)$.

| $x$ | 0 | 0.5 | 1 |
|---|---|---|---|
| $f(x)$ | 0 | 0.247 | 0.841 |

**a**. [3 points] Estimate $f'(1)$. Be sure it is clear how you obtain your answer.

*Solution:* We can estimate the derivative $f'(1)$ with a difference quotient taking $h = -0.5$: $f'(1) \approx \frac{0.841-0.247}{1-0.5} = 1.188$.

**b**. [4 points] Estimate $f''(1)$. Again, be sure that it is clear how you obtain your answer.

*Solution:* We procede similarly to part (a), but need the value of $f'(0.5)$ to complete the difference quotient. Using a central difference for $f'(0.5)$ we have $f'(0.5) \approx \frac{0.841-1}{1} = 0.841$, so that

$$f''(1) \approx \frac{f'(1) - f'(0.5)}{1 - 0.5} = \frac{1.188 - 0.841}{0.5} = 0.694.$$

Alternately, if we use $h = -0.5$ for $f'(0.5)$ as well as for $f'(1)$, we have $f'(0.5) \approx \frac{.247-0}{0.5-0} = 0.494$, so that

$$f''(1) \approx \frac{f'(1) - f'(0.5)}{1 - 0.5} = \frac{1.188 - 0.494}{1 - 0.5} = 1.388.$$

**c**. [3 points] Estimate $f(1.25)$, being sure your work is clear.

*Solution:* We know that $f(1) = 0.841$ and have $f'(1) \approx 1.188$. We can therefore estimate that

$$f(1.25) \approx 0.841 + (0.25)(1.188) = 1.138.$$

**d**. [2 points] Based on your work in (a) and (b), is your estimate in (c) an over- or under-estimate? Explain.

*Solution:* Because $f''(1) \approx 1.388$, we expect that the actual slope between $x = 1$ and $x = 1.25$ is more than our value of 1.388, and thus that this is an underestimate for the actual value of $f(1.25)$.