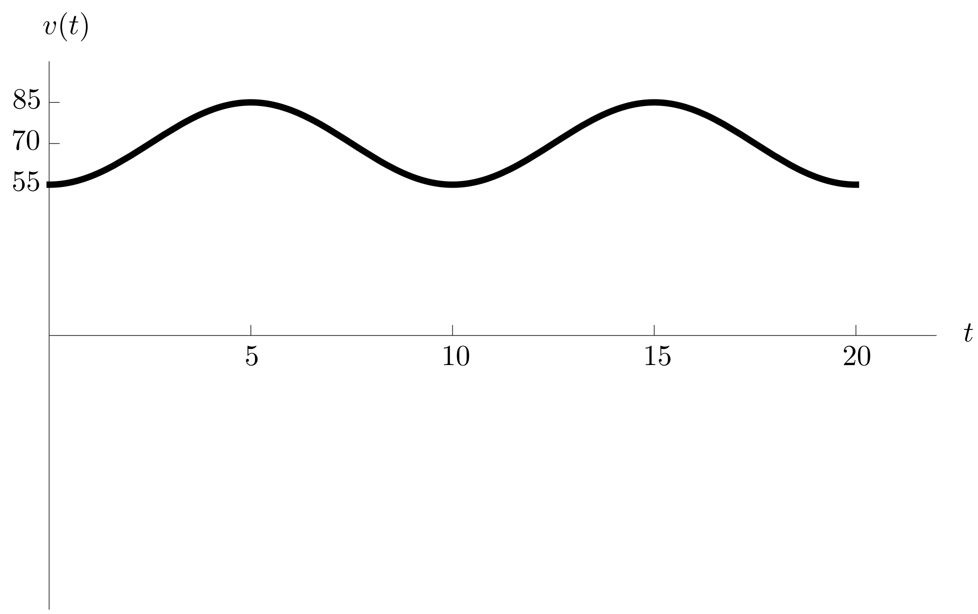
3. [12 points] Suppose that when you merge onto the highway the blue car in front of you is moving at 55 mph. Immediately after you merge, the driver of the blue car speeds up until, after five minutes, it is going 85 mph. Then, during the next five minutes it slows down to 55 mph again. This process then repeats over the following 10 minutes, with the blue car speeding up to 85 mph and then decreasing to 55 mph again.

a. [6 points] Assuming the speed of the blue car follows a sinusoidal pattern, on the axes below draw a well-labeled sketch of two periods of a function $v(t)$ which outputs the speed of the car t minutes after you merge onto the highway.



b. [6 points] Find a possible formula for $v(t)$ from part (a). What are the period and amplitude of $v(t)$?

$v(t) =$ $-15\cos(\frac{\pi}{5}t) + 70$

period = **10 min**

amplitude = **15 mph**

Solution: Notice that the oscillation is between 55 and 85 mph, so that the midline value is 70 mph. Further, the oscillation starts at the minimum value, so that the easiest formula for the velocity will be $v(t) = -A\cos(b\,t) + 70$. Here A is the amplitude of the oscillation, which is $(85 - 55)/2 = 15$. Finally, $b = 2\pi/T$, where T is the period, which is given to be 10 minutes. Thus $b = \pi/5$. The period is 10 min, and the amplitude is 15 mph. Because of the periodicity of sine and cosine, equivalent formulae would be $v(t) = 15\sin(\frac{\pi}{5}(t - 2.5)) + 70$ and $v(t) = 15\cos(\frac{\pi}{5}(t - 5)) + 70$.