5. [12 points] A paperback book (definitely not a valuable calculus textbook, of course) is dropped from the top of Dennison hall (which is 40 m high) towards a very large, upward pointing fan. The average velocity of the book between time $t = 0$ and later times is shown in the table of data below (in which t is in seconds and the velocities are in m/s).

between $t = 0$ seconds and $t =$	1	2	3	4	5
average velocity is	-5	-10	-11.67	-9	-7.2

a. [8 points] Fill in the following table of values for the height $h(t)$ of the book (measured in meters). Show how you obtain your values.

t	0	1	2	3	4	5
$h(t)$	40	**35**	**20**	**5**	**4**	**4**

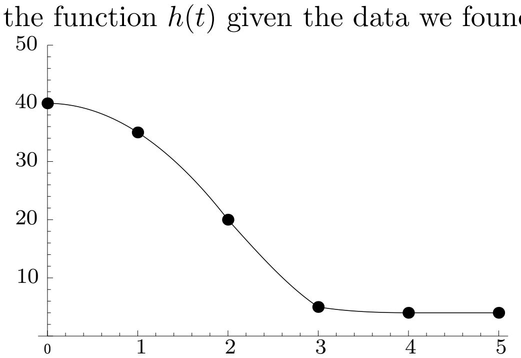
Solution: For each value, we use the definition of average velocity:

$$\text{average velocity on } [0, a] = \frac{h(a) - h(0)}{a}.$$

Thus, the average velocity between $t = 0$ and $t = 1$ gives us $h(1) - 40 = -5$, so $h(1) = 35$. Similarly, between $t = 0$ and $t = 2$ we have $(h(2) - 40)/2 = -10$, so that $h(2) = 20$, etc.

b. [4 points] Based on your work from (a), is $h''(1) > 0$, < 0, or $= 0$? Is $h''(3) > 0$, < 0, or $= 0$? Explain.

Solution: A sketch of the function $h(t)$ given the data we found in (a) is shown below.



We see that $h(t)$ is concave down at $t = 1$ and concave up at $t = 3$. Thus $h''(1) < 0$ and $h''(3) > 0$.

Alternate solution: The average velocity between $t = 0$ and $t = 1$ is -5 and approximates $h'(0.5)$. The average velocity between $t = 1$ and $t = 2$ is $(20-15)/(2-1) = -15 \approx h'(1.5)$. Thus the velocity appears to be decreasing at $t = 1$, so that $h''(1) < 0$. Similarly we have $h'(2.5) \approx -15$ and $h'(3.5) \approx -1$, so $h''(3) > 0$.