

9. [6 points] The population,  $P(t)$ , of China, in billions, can be approximated by

$$P(t) = 1.267(1.007)^t,$$

where  $t$  is the number of years since the start of 2000.

- a. [2 points] Calculate the *continuous growth rate* of  $P(t)$ .

*Solution:* The function  $P(t)$  is an exponential function of the form  $P_0 a^t$  (which can be written as  $P_0 e^{kt}$ ). The continuous growth rate of such an exponential is  $\ln a$  ( $= k$ ). In this case since  $a = 1.007$ , the continuous growth rate is  $\ln 1.007$ .

- b. [4 points] Using the limit definition of the derivative, write an explicit expression for the derivative of  $P(t)$  at the beginning of 2011. You do not need to simplify your expression.

*Solution:* The beginning of 2011 corresponds to  $t = 11$  in this problem. The definition of the derivative of  $P$  at  $t = 11$  is

$$\begin{aligned} P'(11) &= \lim_{h \rightarrow 0} \frac{P(11+h) - P(11)}{h} \\ &= \lim_{h \rightarrow 0} \frac{1.267(1.007)^{11+h} - 1.267(1.007)^{11}}{h}. \end{aligned}$$