5. [12 points] Preparing a pot of soup for dinner, Billy heats the soup to boiling and then removes it from the stove. The function $H(t)$ gives the temperature of the soup in °F as a function of the number of minutes since it was removed from the stove. Assume that $H(0) = 212$ and $H'(0) = -2.7$, and that Billy’s kitchen is carefully air-conditioned to remain at a comfortable 68 °F at all times. Throughout this problem, be sure to include units in your answers, where applicable.

a. [3 points] Approximate $H(1.5)$.

b. [3 points] Five minutes after removing the soup from the stove, Billy remarks to himself: “In the next 30 seconds, I expect the soup to cool by about 0.875 °F.” Since he is both an excellent chef and a student of Math 115, his statement is consistent with the actual value of the derivative of the function $H$. Based on this information, find $H'(5)$, and justify your answer.

c. [3 points] Assume that the concavity of $H(t)$ is the same on its entire domain. Based on your answer to part (b) and the given information, do you expect that the function $H(t)$ is concave up or concave down? Briefly explain your answer.

d. [3 points] Called off on important business, Billy leaves the pot of soup uneaten. Approximate $H'(300)$. (You may use the practical interpretation of $H(t)$, but be sure to explain your answer.)