3. [13 points] A wedge of cheese in Zack's refrigerator has become home to a colony of bacteria. Let $A(t)$ be the surface area of the colony (in cm^2) t days after the expiration date of the cheese.

a. [4 points] For the first 20 days after the expiration date, the surface area of the colony grows exponentially. During this time, it takes the colony 5 days to double. Write a formula for $A(t)$ on the domain $0 \leq t \leq 20$. (Your formula may involve an unknown constant, but be sure to specify what this constant means in terms of bacteria.)

Solution:

$$A(t) = A_0 2^{t/5}, \text{ where } A_0 \text{ is the initial surface area of the colony.}$$

Beginning with $A(t) = A_0 b^t$ we have that the surface area of the bacteria doubles in 5 days, so we set $2A_0 = A_0 b^5$. Then $2^{1/5} = b$.

b. [3 points] How many days does it take for the surface area of the colony to triple? (Your answer does not need to be a whole number.)

Solution:

$$5\frac{\log(3)}{\log(2)} \approx 7.9248 \text{ days}$$

Beginning with our equation from (a) we set $3A_0 = A_0 2^{t/5}$. Taking ln of both sides and simplifying we have $\ln 3/\ln 2 = t/5$.

c. [3 points] Twenty days after the expiration date, the bacteria mysteriously begin to die off. The surface area of the colony on the cheese decreases linearly at a rate of 0.3 cm^2/day starting at $t = 20$, and by $t = 22$ the surface area has fallen to 9 cm^2. Given that $A(t)$ is a continuous function, what was the surface area of the colony on the expiration date of the cheese?

Solution: Working backwards from $t = 22$ we have that the surface area was 0.6 cm^2 more at $t = 20$ than at $t = 22$. This means it was 9.6 cm^2 at $t = 20$ where the exponential growth stopped. Setting

$$9.6 = A_0(2)^{20/5}$$

we have

$$A_0 = 0.6\text{cm}^2$$

d. [3 points] What is $A'(20)$, or is it undefined? Justify your answer with a rough sketch of $A(t)$.

Solution:
It is undefined because $A(t)$ is not differentiable due to corner on its graph at $t = 20$.