5. [12 points] Preparing a pot of soup for dinner, Billy heats the soup to boiling and then removes it from the stove. The function $H(t)$ gives the temperature of the soup in °F as a function of the number of minutes since it was removed from the stove. Assume that $H(0) = 212$ and $H'(0) = -2.7$, and that Billy's kitchen is carefully air-conditioned to remain at a comfortable 68 °F at all times. Throughout this problem, be sure to **include units** in your answers, where applicable.

a. [3 points] Approximate $H(1.5)$.

Solution:

$$H(1.5) \approx 212 + 1.5(-2.7) = 207.95\ ^\circ\text{F}$$

b. [3 points] Five minutes after removing the soup from the stove, Billy remarks to himself: "In the next 30 seconds, I expect the soup to cool by about 0.875 °F." Since he is both an excellent chef and a student of Math 115, his statement is consistent with the actual value of the derivative of the function H. Based on this information, find $H'(5)$, and justify your answer.

Solution: If in 30 seconds the soup will cool by 0.875 °F, in a minute we would expect it to cool by approximately 1.75 °F, so

$$H'(5) = -1.75\ ^\circ\text{F/minute}$$

c. [3 points] Assume that the concavity of $H(t)$ is the same on its entire domain. Based on your answer to part (b) and the given information, do you expect that the function $H(t)$ is concave up or concave down? Briefly explain your answer.

Solution: Between $t = 0$ and $t = 5$ the derivative has increased from approximately -2.7 to approximately -1.75, so $H(t)$ should be concave up.

d. [3 points] Called off on important business, Billy leaves the pot of soup uneaten. Approximate $H'(300)$. (You may use the practical interpretation of $H(t)$, but be sure to explain your answer.)

Solution:

$$H'(200) \approx 0\ ^\circ\text{F/minute.}$$

The function $H(t)$ is concave up and decreasing, so after a long period of time, the slope of the graph should approach zero.