

7. [9 points] Enjoying breakfast outdoors in a coastal Mediterranean town, Tommy notices a ship that is anchored offshore. The ship is stationed above a reef which lies below the surface of the water, and a series of waves causes its height to oscillate sinusoidally with a period of 6 seconds. When Tommy begins observing, the hull of the ship is at its highest point, 20 feet above the reef. After 1.5 seconds, the hull is 11 feet above the reef.

- a. [6 points] Write a function $h(t)$ that gives the height of the ship's hull above the reef t seconds after Tommy begins observing.

Solution:

$$h(t) = 9 \cos\left(\frac{\pi}{3}t\right) + 11.$$

The function starts out at its maximum, so we will use cosine with no horizontal shift making our formula $h(t) = A \cos(Bt) + C$. The period is given to be 6. This means $B = 2\pi/6 = \pi/3$. When $t = 1.5$, we have

$$11 = h(1.5) = A \cos\left(\frac{\pi}{3} \cdot 1.5\right) + C = C$$

and so when $t = 0$ we have

$$20 = h(0) = A + C = A + 11.$$

Solving this, we have $A = 9$.

- b. [3 points] According to your function, will the hull of the ship hit the reef? Explain.

Solution: No. At its lowest point at $t = 3$, $h(3) = 2$, so the ship remains two feet above the reef.