7. [9 points] Enjoying breakfast outdoors in a coastal Mediterranean town, Tommy notices a ship that is anchored offshore. The ship is stationed above a reef which lies below the surface of the water, and a series of waves causes its height to oscillate sinusoidally with a period of 6 seconds. When Tommy begins observing, the hull of the ship is at its highest point, 20 feet above the reef. After 1.5 seconds, the hull is 11 feet above the reef.

a. [6 points] Write a function $h(t)$ that gives the height of the ship’s hull above the reef $t$ seconds after Tommy begins observing.

\[
\begin{align*}
\text{Solution:} \\
\quad & h(t) = 9 \cos \left( \frac{\pi}{3}t \right) + 11.
\end{align*}
\]

The function starts out at its maximum, so we will use cosine with no horizontal shift making our formula $h(t) = A \cos (Bt) + C$. The period is given to be 6. This means $B = 2\pi/6 = \pi/3$. When $t = 1.5$, we have

\[
11 = h(1.5) = A \cos \left( \frac{\pi}{3} \cdot 1.5 \right) + C = C
\]

and so when $t = 0$ we have

\[
20 = h(0) = A + C = A + 11.
\]

Solving this, we have $A = 9$.

b. [3 points] According to your function, will the hull of the ship hit the reef? Explain.

\[
\begin{align*}
\text{Solution:} & \quad \text{No. At it’s lowest point at } t = 3, h(3) = 2, \text{ so the ship remains two feet above the reef.}
\end{align*}
\]