8. [9 points] A certain company's revenue R (in thousands of dollars) is given as a function of the amount of money a (in thousands of dollars) they spend on advertising by $R = f(a)$. Suppose that f is invertible.

a. [2 points] Which of the following is a valid interpretation of the equation $(f^{-1})'(75) = 0.5$? Circle one option.

- If the company spends \$75,000 more on advertising, their revenue will increase by about \$500.
- If the company increases their advertising expenditure from \$75,000 to \$76,000, their revenue will increase by about \$500.
- If the company wants a revenue of \$75,000, they should spend about \$500 on advertising.
- If the company wants to increase their revenue from \$75,000 to \$76,000, they should spend about \$500 more on advertising.

Solution: The last option.

b. [2 points] The company plans to spend about \$100,000 on advertising. If $f'(100) = 0.5$, should the company spend more or less than \$100,000 on advertising? Justify your answer.

Solution:
They should spend less on advertising, because if they increase their advertising expenditure by \$1000, they will only gain about \$500 in revenue.

c. [5 points] The company's financial advisor claims that he has a formula for the dependence of revenue on advertising expenditure, and it is

$$f(a) = a\ln(a+1).$$

Using this formula, write the *limit definition* of $f'(100)$. You do not need to simplify or evaluate.

Solution:

$$f'(100) = \lim_{h\to 0} \frac{(100+h)\ln(100+h+1) - 100\ln(101)}{h}$$