8. [9 points] A certain company’s revenue $R$ (in thousands of dollars) is given as a function of the amount of money $a$ (in thousands of dollars) they spend on advertising by $R = f(a)$. Suppose that $f$ is invertible.

a. [2 points] Which of the following is a valid interpretation of the equation $(f^{-1})'(75) = 0.5$? Circle one option.

- If the company spends $75,000 more on advertising, their revenue will increase by about $500.
- If the company increases their advertising expenditure from $75,000 to $76,000, their revenue will increase by about $500.
- If the company wants a revenue of $75,000, they should spend about $500 on advertising.
- If the company wants to increase their revenue from $75,000 to $76,000, they should spend about $500 more on advertising.

**Solution:** The last option.

b. [2 points] The company plans to spend about $100,000 on advertising. If $f'(100) = 0.5$, should the company spend more or less than $100,000 on advertising? Justify your answer.

**Solution:**
They should spend less on advertising, because if they increase their advertising expenditure by $1000, they will only gain about $500 in revenue.

c. [5 points] The company’s financial advisor claims that he has a formula for the dependence of revenue on advertising expenditure, and it is

$$f(a) = a \ln(a + 1).$$

Using this formula, write the limit definition of $f'(100)$. You do not need to simplify or evaluate.

**Solution:**

$$f'(100) = \lim_{h \to 0} \frac{(100 + h) \ln(100 + h + 1) - 100 \ln(101)}{h}$$