3. [14 points] Laura and Eddie are co-owners of a caffeinated soap factory. Let $M(x)$ denote the mass, in grams, of caffeine in a bar of soap that causes a typical customer's bloodstream caffeine content to be x mg.

a. [4 points] Assuming that $M(x)$ is an invertible function, give a practical interpretation of the statement $M^{-1}(2) = 12$.

Solution: If a bar of soap contains 2 grams of caffeine, it will cause a typical caffeine customer's bloodstream content to be 12 mg.

b. [4 points] Under the same assumption, give a practical interpretation of the statement $M'(13) = 0.7$.

Solution: If we compare a bar of soap that causes a typical customer's bloodstream caffeine content to be 13 mg with one that causes a typical customer's bloodstream caffeine content to be 14 mg, we expect the second bar to contain approximately .7 more grams of caffeine.

c. [6 points]
Laura and Eddie know that $M(x)$ is either a linear or an exponential function, but they aren't sure which. From experimenting, they know that $M(12) = 2$ and $M(14) = 4$. They need more data to determine which is correct. For each of the following hypothetical experimental results, circle EXPONENTIAL if the result shows that $M(x)$ could be exponential, circle LINEAR if the result shows that $M(x)$ could be linear, or circle EITHER if the result does not rule out either possibility. Assume Laura and Eddie's equipment gives experimental evidence which is accurate to within .1 mg.

i. Laura and Eddie discover that $M(x)$ is an invertible function.

EXPONENTIAL LINEAR [EITHER]

ii. Laura and Eddie discover that $M'(17.2) = M'(18.3)$.

EXPONENTIAL [LINEAR] EITHER

iii. Laura and Eddie discover that when there are 7 grams of caffeine in the soap, the caffeine level in a typical customer's bloodstream is roughly 15.6 mg.

[EXPONENTIAL] LINEAR EITHER