

4. [10 points] A motorcyclist heads north from an intersection after a stoplight turns green. The table below records the data on the motorcyclist's speedometer, measuring her velocity, $v(t)$, in feet per second, t seconds after the stoplight turns green. Assume that the motorcyclist does not slow down at any point during the interval of time we are measuring.

t	0	2	4	6
$v(t)$	0	5	15	30

- a. [3 points] Recall that the acceleration function, $a(t)$, is the derivative of the velocity function. Use the table to estimate $a(2)$. Include units.

Solution: We have $a(2) \approx \frac{v(4)-v(2)}{2} = 5$ feet per second per second. Alternatively, $a(2) \approx \frac{v(2)-v(0)}{2-0} = 2.5$ feet per second per second, or we could take the average of these two estimates, 3.75 feet per second per second (or ft/s^2).

- b. [3 points] The “jerk” $j(t)$ of the motorcycle is the derivative of the acceleration function. Use the table to estimate $j(2)$. Include units.

Solution: We can fill in the table with new approximations for $a(t)$ using the same logic as in the previous problem. (Your approximations may be different.)

t	0	2	4
$v(t)$	0	5	15
$a(t) \approx$	2.5	5	7.5

Now $j(2) \approx \frac{a(4)-a(2)}{4-2} \approx 1.25$ feet per second per second per second (or ft/s^3). If you used different estimates for $a(t)$, your answer could be different.

- c. [4 points] Given everything we know about the motorcyclist, can we definitely conclude that $a(4) \leq 8$? If you answer YES, then explain your reasoning. If you answer NO, then sketch a graph of a velocity function $v(t)$ which is consistent with all the information in this problem, but which has $a(4) > 8$.

Solution: No, we cannot conclude that. The graph of $v(t)$ might look like this:

