6. [13 points] Suppose \( n(x) = (x + \frac{1}{2})e^x \).

a. [4 points] Using the limit definition of the derivative, write an explicit expression for \( n'(2) \). Your expression should not contain the letter “\( n \)”. Do not try to evaluate your expression.

\[
\text{Solution:} \quad n'(2) = \lim_{h \to 0} \frac{(2 + h + \frac{1}{2})e^{2+h} - (2 + \frac{1}{2})e^2}{h}
\]

The derivative of \( n(x) \) is \( n'(x) = (x + \frac{3}{2})e^x \).

b. [3 points] Using the given formula for \( n'(x) \), write an equation for the tangent line to the graph of \( n(x) \) at \( x = 2 \).

\[
\text{Solution:} \quad \text{The slope is } (2 + \frac{3}{2})e^2, \text{ and the y-coordinate when } x = 2 \text{ is } (2 + \frac{1}{2})e^2, \text{ so using the point-slope formula for a line, we get the equation}
\]

\[
y - (2 + \frac{1}{2})e^2 = (2 + \frac{3}{2})e^2(x - 2)
\]

c. [3 points] Write an equation for the tangent line to the graph of \( n(x) \) at \( x = a \) where \( a \) is an unknown constant.

\[
\text{Solution:} \quad \text{Using the same logic, we get}
\]

\[
y - (a + \frac{1}{2})e^a = (a + \frac{3}{2})e^a(x - a)
\]

d. [3 points] Using your answer from (c), find a value of \( a \) so that the tangent line to the graph of \( n(x) \) at \( x = a \) passes through the origin.

\[
\text{Solution:} \quad \text{We want the line from the previous part of the problem to pass through } (0, 0), \text{ so we have the equation}
\]

\[
-(a + \frac{1}{2})e^a = (a + \frac{3}{2})e^a(-a).
\]

After dividing out the term \( e^a \), this becomes a quadratic equation in \( a \):

\[
a^2 + \frac{1}{2}a - \frac{1}{2} = 0
\]

Factoring or using the quadratic formula, we conclude that \( a = -1 \) or \( a = \frac{1}{2} \).