6. [13 points] Suppose $n(x) = (x + \frac{1}{2})e^x$.

a. [4 points] Using the limit definition of the derivative, write an explicit expression for $n'(2)$. Your expression should not contain the letter "n". Do not try to evaluate your expression.

Solution:

$$n'(2) = \lim_{h\to 0} \frac{(2 + h + \frac{1}{2})e^{2+h} - (2 + \frac{1}{2})e^2}{h}$$

The derivative of $n(x)$ is $n'(x) = (x + \frac{3}{2})e^x$.

b. [3 points] Using the given formula for $n'(x)$, write an equation for the tangent line to the graph of $n(x)$ at $x = 2$.

Solution: The slope is $(2 + \frac{3}{2})e^2$, and the y-coordinate when $x = 2$ is $(2 + \frac{1}{2})e^2$, so using the point-slope formula for a line, we get the equation

$$y - (2 + \frac{1}{2})e^2 = (2 + \frac{3}{2})e^2(x - 2)$$

c. [3 points] Write an equation for the tangent line to the graph of $n(x)$ at $x = a$ where a is an unknown constant.

Solution: Using the same logic, we get

$$y - (a + \frac{1}{2})e^a = (a + \frac{3}{2})e^a(x - a)$$

d. [3 points] Using your answer from (c), find a value of a so that the tangent line to the graph of $n(x)$ at $x = a$ passes through the origin.

Solution: We want the line from the previous part of the problem to pass through $(0, 0)$, so we have the equation

$$-(a + \frac{1}{2})e^a = (a + \frac{3}{2})e^a(-a).$$

After dividing out the term e^a, this becomes a quadratic equation in a:

$$a^2 + \frac{1}{2}a - \frac{1}{2} = 0$$

Factoring or using the quadratic formula, we conclude that $a = -1$ or $a = \frac{1}{2}$.