9. [7 points] The air in a factory is being filtered so that the quantity of a pollutant, P (in mg/liter), is decreasing exponentially. Suppose t is the time in hours since the factory began filtering the air. Also assume 20% of the pollutant is removed in the first five hours.

a. [2 points] What percentage of the pollutant is left after 10 hours?

Solution: Since 80% of the pollution is left after 5 hours, we see that 80% of 80% of the pollution will be left after 10 hours. This is 64%.

b. [5 points] How long is it before the pollution is reduced by 50%?

Solution: Using the equation

$$P(t) = P_0 a^t,$$

and plugging in the known point $(5, .8P_0)$, we see that

$$.8P_0 = P_0 a^5.$$

So $a = (.8)^{\frac{1}{5}}$ and the formula is

$$P(t) = P_0(.8)^{\frac{t}{5}}.$$

We could also have gotten this formula without algebra, using the definition of decay rate and converting from intervals of 5 hours to intervals of hours.
We want to find a value of t such that $P(t) = .5P_0$. Algebraically, this says that

$$.5P_0 = P_0(.8)^{\frac{t}{5}}.$$

Dividing out P_0 and taking logarithms, we get

$$\ln(.5) = \frac{t}{5}\ln(.8),$$

so $t = 5\frac{\ln(.5)}{\ln(.8)} \approx 15.53$ hours. We could also solve this problem with the initial equation

$$P(t) = P_0 e^{kt}.$$

If you did it that way, you should have gotten $k \approx -.0446$.

10. [5 points] Define a function

$$f(x) = \begin{cases} \dfrac{-x^3 + 5x^2}{x - 5} & x \neq 5, \\ k & x = 5. \end{cases}$$

a. [3 points] Find a value of k so that $f(x)$ is a continuous function for all real numbers x.

Solution: The formula for $f(x)$ when $x \neq 5$ simplifies algebraically to $-x^2$. Since we want $f(x)$ to be continuous when $x = 5$, we need

$$k = \lim_{x \to 5} f(x) = -(5^2) = -25.$$

b. [2 points] For the value of k you found, is $f(x)$ differentiable at $x = 5$? Briefly explain.