

10. [10 points] Throughout this page, give all answers in exact form. Do not use decimal approximations. For example, $x = \frac{1}{3}$ is an exact solution to $3x = 1$, but $x = 0.3333333333$ is not.

Kathy is making hot chocolate one morning while out camping in the cold. She heats it for ten minutes, during which time its temperature increases at a constant rate from 2°C to 80°C . Let $H(t)$ be the temperature, in $^\circ\text{C}$, of the chocolate, t minutes after Kathy begins heating it.

- a. [2 points] Find a formula for $H(t)$ which is valid for $0 < t < 10$.

Answer: $H(t) =$ _____

- b. [5 points] After ten minutes, when the chocolate is 80°C , Kathy turns off her camping stove. The temperature of the chocolate begins to decay exponentially so that its temperature, in $^\circ\text{C}$, decreases by 25% every two minutes. Find a formula for $H(t)$ which is valid for $t \geq 10$.

Answer: $H(t) =$ _____

When Kathy gets home, she discovers that her water bottle is full of ice. From the moment she gets home, it takes 30 minutes for the ice to melt completely. Let $V(t)$ be the volume, in cubic inches, of the ice in Kathy's water bottle, t minutes after she gets home. Until the ice is gone, a formula for V is given by the equation $V(t) = -4 \ln(kt + b)$ for some constants k and b .

- c. [3 points] Find the value of k in terms of b .

Answer: $k =$ _____