4. [9 points] Let $P(v)= \begin{cases}v^{2} \sin \left(\frac{1}{v}\right)-v \sin (2) & \text { if } v \neq 0 \\ 0 & \text { if } v=0 .\end{cases}$
a. [5 points]

Use the limit definition of the derivative to write down an explicit expression for $P^{\prime}(0)$. Your answer should not include the letter $P$.
Do not attempt to evaluate or simplify the limit.
$P^{\prime}(0)=$
b. [4 points] Use your answer to (a) to estimate $P^{\prime}(0)$ to the nearest hundredth. Be sure to include enough clear graphical or numerical evidence to justify your answer.

Answer: $\quad P^{\prime}(0) \approx$ $\qquad$

