1. [11 points] The table below gives several values of a continuous, invertible function $f(x)$. Assume that the domain of both $f(x)$ and $f'(x)$ is the interval $(-\infty, \infty)$.

x	0	3	6	9	12	15	18	21	24
$f(x)$	-7	-3.5	-2	3	4.5	6	7	9	19

a. [3 points] Evaluate each of the following.

(i) $f(f(15))$

Solution: $f(f(15)) = f(6) = -2.$

Answer: $f(f(15)) =$ -2

(ii) $f^{-1}(3)$

Answer: $f^{-1}(3) =$ 9

(iii) $f^{-1}(2f(12))$

Solution: $f^{-1}(2f(12)) = f^{-1}(2(4.5)) = f^{-1}(9) = 21.$

Answer: $f^{-1}(2f(12)) =$ 21

b. [2 points] Compute the average rate of change of f on the interval $3 \le x \le 18$.

Solution: This average rate of change is equal to the difference quotient

$$\frac{f(18) - f(3)}{18 - 3} = \frac{7 - (-3.5)}{15} = \frac{10.5}{15} = \frac{7}{10} = 0.7.$$

Answer: $10.5/15 = 7/10 = 0.7$

c. [2 points] Estimate $f'(19)$.

Solution: We approximate $f'(19)$ by the average rate of change of f on the interval $18 \le x \le 21$.

$$f'(19) \approx \frac{f(21) - f(18)}{21 - 18} = \frac{9 - 7}{3} = \frac{2}{3}.$$

Answer: $f'(19) \approx$ $2/3 \approx 0.67$

d. [2 points] Let $g(x) = f^{-1}(x)$. Estimate $g'(5)$.

Solution: We approximate $g'(5)$ by the average rate of change of $g(x)$ on the interval $4.5 \le x \le 6$.

$$g'(5) \approx \frac{g(6) - g(4.5)}{6 - 4.5} = \frac{f^{-1}(6) - f^{-1}(4.5)}{1.5} = \frac{15 - 12}{1.5} = \frac{3}{1.5} = 2.$$

Answer: $g'(5) \approx$ $3/1.5 = 2$

e. [2 points] Suppose $f'(0) = 2$. Find an equation for the tangent line to the graph of $y = f(x)$ at $x = 0$.

Solution: This is the line with slope $f'(0) = 2$ that passes through the point $(0, f(0)) = (0, -7)$. An equation for this line is $y = 2x - 7$.

Answer: $y = 2x - 7$