10. [10 points] *Throughout this page, give all answers in exact form. Do not use decimal approximations. For example, $x = \frac{1}{3}$ is an exact solution to $3x = 1$, but $x = 0.3333333333$ is not.*
Kathy is making hot chocolate one morning while out camping in the cold. She heats it for ten minutes, during which time its temperature increases at a constant rate from 2°C to 80°C. Let $H(t)$ be the temperature, in °C, of the chocolate, t minutes after Kathy begins heating it.

a. [2 points] Find a formula for $H(t)$ which is valid for $0 < t < 10$.

Solution: The temperature increases at a constant rate, so $H(t)$ is linear with slope equal to the constant average rate of change $\frac{H(10)-H(0)}{10-0} = \frac{80-2}{10} = 7.8$°C/min. Since $H(0) = 2$, we see that $H(t) = 2 + 7.8t$.

Answer: $H(t) =$ $2 + 7.8t$

b. [5 points] After ten minutes, when the chocolate is 80°C, Kathy turns off her camping stove. The temperature of the chocolate begins to decay exponentially so that its temperature, in °C, decreases by 25% every two minutes.
Find a formula for $H(t)$ which is valid for $t \geq 10$.

Solution: Since the temperature decays exponentially, for $t \geq 10$, there are constants c and a so that $H(t) = ca^t$. Because the temperature decreases by 25% every two minutes, $a^2 = 0.75$. (To see this, note that since $H(10) = 80$ and $H(12) = 0.75(80) = 60$, we have $ca^{10} = 80$ and $ca^{12} = 60$. Taking the ratios of the two sides of these equations we find $a^2 = \frac{60}{80} = 0.75$.)
Thus $a = (0.75)^{1/2}$ and we solve for c in the equation $80 = c(0.75)^{(1/2)(10)}$ to find that $c = \frac{80}{(0.75)^5}$. Hence, for $t \geq 10$, we have $H(t) = \frac{80}{(0.75)^5}(0.75)^{0.5t}$. (Note this can also be written as $H(t) = 80(0.75)^{(t-10)/2}$.)

Answer: $H(t) =$ $\frac{80}{(0.75)^5}(0.75)^{0.5t}$ **or** $80(0.75)^{(t-10)/2}$

When Kathy gets home, she discovers that her water bottle is full of ice. From the moment she gets home, it takes 30 minutes for the ice to melt completely. Let $V(t)$ be the volume, in cubic inches, of the ice in Kathy's water bottle, t minutes after she gets home. Until the ice is gone, a formula for V is given by the equation $V(t) = -4\ln(kt+b)$ for some constants k and b.

c. [3 points] Find the value of k in terms of b.

Solution: Since $V(30) = 0$, we solve for k in the equation $-4\ln(k(30) + b = 0$.

$$-4\ln(k(30) + b) = 0$$
$$\ln(30k + b) = 0$$
$$30k + b = 1 (= e^0)$$
$$k = \frac{1-b}{30}$$

Answer: $k =$ $\frac{1-b}{30}$