2. [12 points] A scientist is growing a very large quantity of mold. Initially, the mass of mold grows exponentially, but after many hours, the mass stabilizes at 24 kilograms.
Suppose that t hours after the scientist begins, the mass of mold, in kilograms, can be modeled by the function M defined by the equation

$$M(t) = \begin{cases} 0.41e^{0.72t} & \text{if } 0 \leq t \leq 5 \\ \dfrac{2t^3}{at^b + c} & \text{if } t > 5. \end{cases}$$

a. [4 points] Find the value of k between 0 and 5 so that $M(k) = 1$. Then interpret the equation $M(k) = 1$ in the context of this problem. Use a complete sentence and include units.

Solution: Because we want to find k between 0 and 5, we use the first piece of the formula for M and solve for k in the equation $0.41e^{0.72k} = 1$.

$$\begin{aligned} 0.41e^{0.72k} &= 1 \\ e^{0.72k} &= 1/0.41 \approx 2.439 \\ 0.72k &= \ln(1/0.41) \approx 0.892 \\ k &= \ln(1/0.41)/0.72 \approx 1.238 \end{aligned}$$

Answer: $k =$ $\frac{\ln(1/0.41)}{0.72} \approx 1.238$

Interpretation:

Solution: 1.238 hours after the scientist begins, the mold has a mass of 1 kg.

b. [8 points] Assuming that M is a continuous function of t, determine $\lim_{t\to\infty} M(t)$, and find the values of a, b, and c.

Solution:

$$\frac{2}{a} = 24 \text{ so } a = 1/12 \approx 0.083$$

$$\begin{aligned} 0.41e^{0.72\cdot 5} &= \frac{2 \cdot 5^3}{5^3/12 + c} \\ 0.41e^{3.6} &= \frac{250}{125/12 + c} \\ \frac{125}{12} + c &= \frac{250}{0.41e^{3.6}} \\ c &= \frac{250}{0.41e^{3.6}} - \frac{125}{12} \approx 6.244 \end{aligned}$$

Answers: $\lim_{t\to\infty} M(t) =$ 24 $a =$ $1/12 \approx 0.083$

$b =$ 3 $c =$ $\frac{250}{0.41e^{3.6}} - \frac{125}{12} \approx 6.244$