4. [9 points] Let $P(v) = \begin{cases} v^2 \sin\left(\frac{1}{v}\right) - v\sin(2) & \text{if } v \neq 0 \\ 0 & \text{if } v = 0. \end{cases}$

a. [5 points]
Use the limit definition of the derivative to write down an explicit expression for $P'(0)$.
Your answer should not include the letter P.
Do not attempt to evaluate or simplify the limit.

$$P'(0) = \lim_{h\to 0} \frac{\left((0+h)^2 \sin\left(\frac{1}{0+h}\right) - (0+h)\sin(2)\right) - 0}{h}$$

b. [4 points] Use your answer to (a) to estimate $P'(0)$ to the nearest hundredth.
Be sure to include enough clear graphical or numerical evidence to justify your answer.

Solution: We plug in small values of h approaching 0. Since the difference quotient is an even function of h, we need only check positive values of h (as evenness implies that negative h give precisely the same results).

$h = 0.1$:
$$\frac{0.1^2 \sin(1/0.1) - 0.1\sin(2) - 0}{0.1} \approx -0.964$$

$h = 0.01$:
$$\frac{0.01^2 \sin(1/0.01) - 0.01\sin(2) - 0}{0.01} \approx -0.914$$

$h = 0.001$:
$$\frac{0.001^2 \sin(1/0.001) - 0.001\sin(2) - 0}{0.001} \approx -0.908$$

$h = 0.0001$:
$$\frac{0.0001^2 \sin(1/0.0001) - 0.0001\sin(2) - 0}{0.0001} \approx -0.909$$

We see at this point that the numbers seem to have stabilized to the nearest hundredth at -0.91.

Answer: $P'(0) \approx$ -0.91