4. [9 points] Let \( P(v) = \begin{cases} v^2 \sin \left( \frac{1}{v} \right) - v \sin(2) & \text{if } v \neq 0 \\ 0 & \text{if } v = 0. \end{cases} \)

a. [5 points]
Use the limit definition of the derivative to write down an explicit expression for \( P'(0) \). Your answer should not include the letter \( P \). Do not attempt to evaluate or simplify the limit.

\[
P'(0) = \lim_{h \to 0} \frac{(0 + h)^2 \sin \left( \frac{1}{0+h} \right) - (0 + h) \sin(2)}{h}
\]

b. [4 points] Use your answer to (a) to estimate \( P'(0) \) to the nearest hundredth. Be sure to include enough clear graphical or numerical evidence to justify your answer.

\[\text{Solution:}\quad \text{We plug in small values of } h \text{ approaching 0. Since the difference quotient is an even function of } h, \text{ we need only check positive values of } h \text{ (as evenness implies that negative } h \text{ give precisely the same results).}
\]

\[
\begin{array}{ll}
h = 0.1: & \frac{0.1^2 \sin(1/0.1) - 0.1 \sin(2) - 0}{0.1} \approx -0.964 \\
h = 0.01: & \frac{0.01^2 \sin(1/0.01) - 0.01 \sin(2) - 0}{0.01} \approx -0.914 \\
h = 0.001: & \frac{0.001^2 \sin(1/0.001) - 0.001 \sin(2) - 0}{0.001} \approx -0.908 \\
h = 0.0001: & \frac{0.0001^2 \sin(1/0.0001) - 0.0001 \sin(2) - 0}{0.0001} \approx -0.909 \\
\end{array}
\]

We see at this point that the numbers seem to have stabilized to the nearest hundredth at \(-0.91\).

\[\text{Answer: } P'(0) \approx -0.91\]