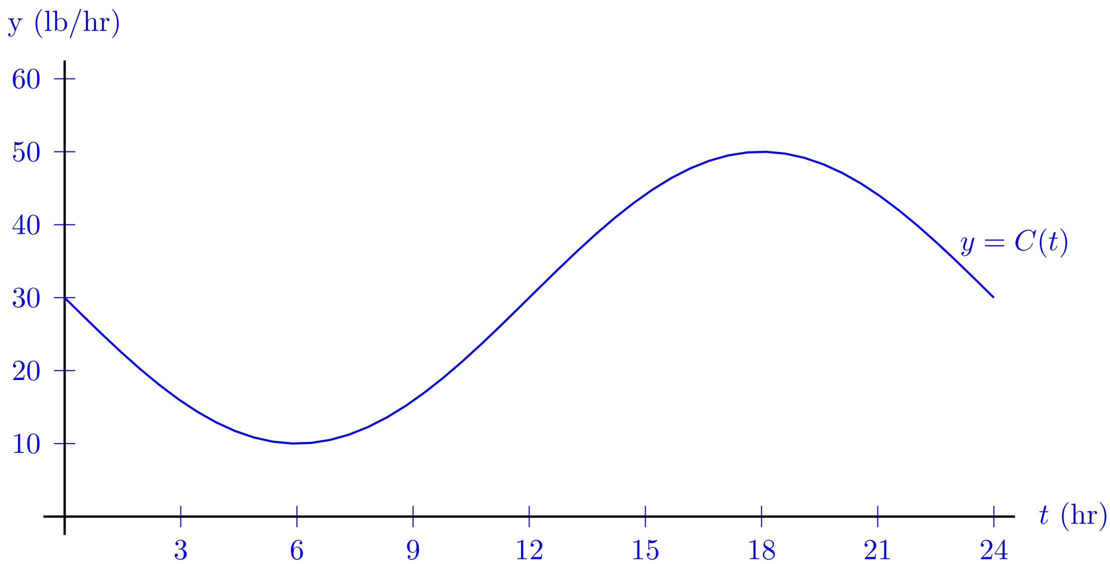
5. [13 points] Jordan owns a 24-hour coffee shop. The coffee brewing rate (or CBR) at Jordan's coffee shop varies throughout the day. The CBR is highest at 6 AM, when coffee is brewed at a rate of 50 pounds of coffee per hour. It is lowest at 6 PM, when coffee is brewed at a rate of only 10 pounds of coffee per hour. Suppose that t hours after noon, the CBR, in pounds of coffee per hour, of Jordan's coffee shop can be modeled by a sinusoidal function $C(t)$ with period 24 hours.

a. [4 points] On the axes provided below, sketch a well-labeled graph of $C(t)$ for $0 \le t \le 24$.



b. [4 points] Find a formula for $C(t)$.

Answer: $C(t) =$ $-20\sin\left(\frac{\pi}{12}t\right) + 30$

c. [5 points] For how many hours each day is the CBR of Jordan's shop at least 40 pounds of coffee per hour? *Remember to show your work.*

Solution: We wish to find the two solutions to $C(t) = 40$ for $0 \le t \le 24$. We start by finding any solution:

$$\begin{aligned} -20\sin\left(\frac{\pi}{12}t\right) + 30 &= 40 \\ \sin\left(\frac{\pi}{12}t\right) &= -0.5 \\ \frac{\pi}{12}t &= \arcsin(-0.5) = -\pi/6 \\ t &= -2. \end{aligned}$$

One of the solutions we want is therefore $t = -2 + 24 = 22$, and by symmetry around the peak at 18, the other is $t = 14$.
Therefore, the CBR is at least 40 for the 8 hours between $t = 14$ and $t = 22$.

Answer: **8 hours**