5. [13 points] Jordan owns a 24-hour coffee shop. The coffee brewing rate (or CBR) at Jordan’s coffee shop varies throughout the day. The CBR is highest at 6 AM, when coffee is brewed at a rate of 50 pounds of coffee per hour. It is lowest at 6 PM, when coffee is brewed at a rate of only 10 pounds of coffee per hour. Suppose that \( t \) hours after noon, the CBR, in pounds of coffee per hour, of Jordan’s coffee shop can be modeled by a sinusoidal function \( C(t) \) with period 24 hours.

a. [4 points] On the axes provided below, sketch a well-labeled graph of \( C(t) \) for \( 0 \leq t \leq 24 \).

![Graph of C(t) from 0 to 24 hours](Image)

b. [4 points] Find a formula for \( C(t) \).

**Answer:**

\[ C(t) = \frac{-20 \sin \left( \frac{\pi}{12} t \right) + 30}{-20 \sin \left( \frac{\pi}{12} t \right) + 30} \]

c. [5 points] For how many hours each day is the CBR of Jordan’s shop at least 40 pounds of coffee per hour? Remember to show your work.

**Solution:** We wish to find the two solutions to \( C(t) = 40 \) for \( 0 \leq t \leq 24 \). We start by finding any solution:

\[ -20 \sin \left( \frac{\pi}{12} t \right) + 30 = 40 \]
\[ \sin \left( \frac{\pi}{12} t \right) = -0.5 \]
\[ \frac{\pi}{12} t = \arcsin(-0.5) = -\frac{\pi}{6} \]
\[ t = -2. \]

One of the solutions we want is therefore \( t = -2 + 24 = 22 \), and by symmetry around the peak at 18, the other is \( t = 14 \).

Therefore, the CBR is at least 40 for the 8 hours between \( t = 14 \) and \( t = 22 \).

**Answer:** 8 hours