

7. [15 points] During the winter, the town of Waterville uses salt to keep the roads from freezing. Let $S = f(T)$ be the amount of salt, in tons, used on the roads of Waterville on a day when the average temperature is T °F. Let $C = g(S)$ be the cost, in thousands of dollars, of S tons of salt. Assume that both f and g are invertible functions that are differentiable everywhere.

- a. [3 points] Interpret the equation $f^{-1}(4) = 9$ in the context of this problem. Use a complete sentence and include units.

Solution: On a day when Waterville uses 4 tons of salt on the roads, the average temperature is 9 °F.

- b. [3 points] Interpret the equation $g(f(7)) = 2$ in the context of this problem. Use a complete sentence and include units.

Solution: On a day when the average temperature is 7 °F, the salt used on Waterville's roads costs \$2000.

- c. [2 points] Yesterday, the average temperature in Waterville was w °F. Give a single mathematical expression equal to the average temperature, in °F, on a day when Waterville uses twice as much salt on the roads as it did yesterday.

Answer: _____ $f^{-1}(2f(w))$

- d. [4 points] Give a single mathematical equality involving the derivative of f which supports the following claim:
On a day when the average temperature is 3°F, Waterville uses approximately 0.12 tons less salt on the roads than on a day when the average temperature is 1°F.

Answer: _____ $f'(1) = -0.06$

- e. [3 points] In the equation $(g^{-1})'(8) = 5$, what are the units on 8 and 5?

Answer: Units on 8 are _____ **thousands of dollars**

Answer: Units on 5 are _____ **tons of salt per thousand dollars**