1. [8 points] The table below gives several values of the continuous, invertible, differentiable functions \( f(x) \) and \( g(x) \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>1.8</th>
<th>1.9</th>
<th>2</th>
<th>2.1</th>
<th>2.2</th>
<th>2.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>2.5</td>
<td>2.35</td>
<td>2.2</td>
<td>2.</td>
<td>1.8</td>
<td>1.7</td>
</tr>
<tr>
<td>( g(x) )</td>
<td>1.6</td>
<td>1.75</td>
<td>1.8</td>
<td>1.9</td>
<td>2</td>
<td>2.2</td>
</tr>
</tbody>
</table>

a. [2 points] Compute \( f(g^{-1}(2)) \).

**Solution:**

\[
f(g^{-1}(2)) = f(2.2) = 1.8.
\]

**Answer:** \( f(g^{-1}(2)) = 1.8 \)

b. [2 points] Estimate \( f'(2) \).

**Solution:** We approximate using difference quotients. Using the average rate of change between \( x = 1.9 \) and \( x = 2 \) we have \( f'(2) \approx \frac{2.2 - 2.35}{2 - 1.9} = -1.5 \), and using the average rate of change between \( x = 2 \) and \( x = 2.1 \) we have \( f'(2) \approx \frac{0.3 - 2.2}{0.2} = -2 \). Averaging these two estimates, we find the estimate \( f'(2) \approx -1.75 \).

**Answer:** \( f'(2) \approx -1.75 \)

c. [2 points] Let \( j(x) = g^{-1}(x) \). Estimate \( j'(1.9) \).

**Solution:** A table of values for \( j(x) \) is given by:

<table>
<thead>
<tr>
<th>( x )</th>
<th>1.6</th>
<th>1.75</th>
<th>1.8</th>
<th>1.9</th>
<th>2</th>
<th>2.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( j(x) )</td>
<td>1.8</td>
<td>1.9</td>
<td>2</td>
<td>2.1</td>
<td>2.2</td>
<td>2.3</td>
</tr>
</tbody>
</table>

Estimating \( j'(1.9) \) using the average rate of change between \( x = 1.9 \) and \( x = 2 \) we find \( j'(2) \approx \frac{2.2 - 2.1}{2 - 1.9} = 1 \). (We obtain the same estimate using the interval from \( x = 1.8 \) to \( x = 1.9 \).)

**Answer:** \( j'(1.9) \approx 1 \)

d. [2 points] Suppose \( p(x) \) is a function whose derivative is given by \( p'(x) = \ln(x^3 + 11) \). Compute \( p'(2) \).

**Solution:**

\[
p'(2) = p'(2) = \ln((2.2)^3 + 11) = \ln(21.648) \approx 3.0749
\]

**Answer:** \( p'(2) = \ln(21.648) \)

2. [6 points] Suppose \( a \) and \( b \) are constants with \( a > 3 \) and \( b > 0 \), and let \( h(t) = a^{-bt} \).

a. [3 points] Find constants \( P_0 \) and \( k \) so that \( h(t) = P_0 e^{kt} \). (Your answers may involve the constants \( a \) and/or \( b \).)

**Solution:** If \( P_0 e^{kt} = a^{-bt} \) for all \( t \), then \( P_0 = 1 \) and \( a^{-b} = e^k \), so \( k = \ln(a^{-b}) = -b \ln(a) \).

Alternatively, we can directly rewrite the original formula as \( h(t) = a^{-bt} = (e^{\ln a})^{-bt} = e^{-b \ln(a)t} \).

**Answer:** \( P_0 = 1 \) and \( k = -b \ln(a) \)

b. [3 points] Circle all the statements below that must be true about the function \( h(t) \). If none of the statements must be true, circle NONE OF THESE.

i. The domain of \( h(t) \) is the interval \((-\infty, \infty)\).

ii. The range of \( h(t) \) is the interval \((-\infty, \infty)\).

iii. \( h(t) \) is an increasing function on its domain.

iv. \( h(t) \) is concave up on its domain.

v. \( t = 0 \) is a vertical asymptote of the graph of \( h(t) \).

vi. \( \lim_{t \to \infty} h(t) = 0 \).

vii. \( \lim_{t \to -\infty} h(t) = 0 \).

**Solution:** Since \( a > 1 \) and \( b > 0 \), \( h(t) \) is a decreasing exponential function.

NONE OF THESE