1. [8 points] The table below gives several values of the continuous, invertible, differentiable functions $f(x)$ and $g(x)$.

x	1.8	1.9	2	2.1	2.2	2.3
$f(x)$	2.5	2.35	2.2	2	1.8	1.7
$g(x)$	1.6	1.75	1.8	1.9	2	2.2

a. [2 points] Compute $f(g^{-1}(2))$.

Solution:
$$f(g^{-1}(2)) = f(2.2) = 1.8\ .$$

Answer: $f(g^{-1}(2)) =$ 1.8

b. [2 points] Estimate $f'(2)$.

Solution: We approximate using difference quotients. Using the average rate of change between $x = 1.9$ and $x = 2$ we have $f'(2) \approx \frac{2.2-2.35}{0.1} = -1.5$, and using the average rate of change between $x = 2$ and $x = 2.1$ we have $f'(2) \approx \frac{2-2.2}{0.1} = -2$. Averaging these two estimates, we find the estimate $f'(2) \approx -1.75$.

Answer: $f'(2) \approx$ -1.75

c. [2 points] Let $j(x) = g^{-1}(x)$. Estimate $j'(1.9)$.

Solution: A table of values for $j(x)$ is given by

x	1.6	1.75	1.8	1.9	2	2.2
$j(x)$	1.8	1.9	2	2.1	2.2	2.3

Estimating $j'(1.9)$ using the average rate of change between $x = 1.9$ and and $x = 2$ we find $j'(2) \approx \frac{2.2-2.1}{2-1.9} = 1$. (We obtain the same estimate using the interval from $x = 1.8$ to $x = 1.9$.)

Answer: $j'(1.9) \approx$ 1

d. [2 points] Suppose $p(x)$ is a function whose derivative is given by $p'(x) = \ln(x^3 + 11)$. Compute $p'(f(2))$.

Solution: $p'(f(2)) = p'(2.2) = \ln((2.2)^3 + 11) = \ln(21.648) \approx 3.0749$

Answer: $p'(f(2)) =$ $\ln(21.648)$

2. [6 points] Suppose a and b are constants with $a > 3$ and $b > 0$, and let $h(t) = a^{-bt}$.

a. [3 points] Find constants P_0 and k so that $h(t) = P_0 e^{kt}$. (Your answers may involve the constants a and/or b.)

Solution: If $P_0 e^{kt} = a^{-bt}$ for all t, then $P_0 = 1$ and $a^{-b} = e^k$, so $k = \ln(a^{-b}) = -b\ln(a)$. Alternatively, we can directly rewrite the original formula as $h(t) = a^{-bt} = (e^{\ln a})^{-bt} = e^{-b\ln(a)t}$.

Answer: $P_0 =$ 1 and $k =$ $-b\ln(a)$

b. [3 points] Circle <u>all</u> the statements below that <u>must</u> be true about the function $h(t)$. If none of the statements must be true, circle NONE OF THESE.

Solution: Since $a > 1$ and $b > 0$, $h(t)$ is a decreasing exponential function.

i. The domain of $h(t)$ is the interval $(-\infty, \infty)$.

ii. The range of $h(t)$ is the interval $(-\infty, \infty)$.

iii. $h(t)$ is an increasing function on its domain.

iv. $h(t)$ is concave up on its domain.

v. $t = 0$ is a vertical asymptote of the graph of $h(t)$.

vi. $\lim_{t\to\infty} h(t) = 0$.

vii. $\lim_{t\to-\infty} h(t) = 0$.

NONE OF THESE