1. [8 points] The table below gives several values of the continuous, invertible, differentiable functions \( f(x) \) and \( g(x) \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>1.8</th>
<th>1.9</th>
<th>2</th>
<th>2.1</th>
<th>2.2</th>
<th>2.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>2.5</td>
<td>2.35</td>
<td>2.2</td>
<td>2.1</td>
<td>2.2</td>
<td>2.3</td>
</tr>
<tr>
<td>( g(x) )</td>
<td>1.6</td>
<td>1.75</td>
<td>1.8</td>
<td>1.9</td>
<td>2</td>
<td>2.2</td>
</tr>
</tbody>
</table>

a. [2 points] Compute \( f(g^{-1}(2)) \).

**Solution:**

\[ f(g^{-1}(2)) = f(2.2) = 1.8. \]

**Answer:** \( f(g^{-1}(2)) = 1.8 \)

b. [2 points] Estimate \( f'(2) \).

**Solution:** We approximate using difference quotients. Using the average rate of change between \( x = 1.9 \) and \( x = 2 \) we have \( f'(2) \approx \frac{2.2 - 2.15}{2 - 1.9} = -1.5 \), and using the average rate of change between \( x = 2 \) and \( x = 2.1 \), we have \( f'(2) \approx \frac{0.1 - 2}{0.1} = -2 \). Averaging these two estimates, we find the estimate \( f'(2) \approx -1.75 \).

**Answer:** \( f'(2) \approx -1.75 \)

c. [2 points] Let \( j(x) = g^{-1}(x) \). Estimate \( j'(1.9) \).

**Solution:** A table of values for \( j(x) \) is given by

<table>
<thead>
<tr>
<th>( x )</th>
<th>1.6</th>
<th>1.75</th>
<th>1.8</th>
<th>1.9</th>
<th>2</th>
<th>2.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( j(x) )</td>
<td>1.8</td>
<td>1.9</td>
<td>2</td>
<td>2.1</td>
<td>2.2</td>
<td>2.3</td>
</tr>
</tbody>
</table>

Estimating \( j'(1.9) \) using the average rate of change between \( x = 1.9 \) and \( x = 2 \) we find \( j'(2) \approx \frac{2.2 - 2.1}{2 - 1.9} = 1 \). (We obtain the same estimate using the interval from \( x = 1.8 \) to \( x = 1.9 \)).

**Answer:** \( j'(1.9) \approx 1 \)

d. [2 points] Suppose \( p(x) \) is a function whose derivative is given by \( p'(x) = \ln(x^3 + 11) \). Compute \( p'(2) \).

**Solution:**

\[ p'(f(2)) = p'(2.2) = \ln((2.2)^3 + 11) = \ln(21.648) \approx 3.0749 \]

**Answer:** \( p'(f(2)) = \ln(21.648) \)

2. [6 points] Suppose \( a \) and \( b \) are constants with \( a > 3 \) and \( b > 0 \), and let \( h(t) = a^{-bt} \).

a. [3 points] Find constants \( P_0 \) and \( k \) so that \( h(t) = P_0 e^{kt} \). (Your answers may involve the constants \( a \) and/or \( b \)).

**Solution:** If \( P_0 e^{kt} = a^{b^t} \) for all \( t \), then \( P_0 = 1 \) and \( a^{-b} = e^k \), so \( k = \ln(a^{-b}) = -b \ln(a) \).

Alternatively, we can directly rewrite the original formula as \( h(t) = a^{-bt} = (e^{\ln a})^{bt} = e^{-b \ln(a) t} \).

**Answer:** \( P_0 = 1 \) and \( k = -b \ln(a) \)

b. [3 points] Circle all the statements below that **must** be true about the function \( h(t) \). If none of the statements must be true, circle **NONE OF THESE**.

i. The domain of \( h(t) \) is the interval \((-\infty, \infty)\).

ii. The range of \( h(t) \) is the interval \((-\infty, \infty)\).

iii. \( h(t) \) is an increasing function on its domain.

iv. \( h(t) \) is concave up on its domain.

v. \( t = 0 \) is a vertical asymptote of the graph of \( h(t) \).

vi. \( \lim_{t \to \infty} h(t) = 0 \).

vii. \( \lim_{t \to -\infty} h(t) = 0 \).

**Answer:** NONE OF THESE