

3. [10 points] Elphaba the squirrel has been involved in some questionable activity of late and hence is being very cautious. She has made eye contact with a human standing near her multiple times and is getting anxious that the human is observing her. Let $f(x)$ be Elphaba's anxiety (in "anxious units") after making eye contact with the human for a total of x seconds. Elphaba will panic and run when her anxiety reaches 100 anxious units.

From across the room, the human, Erin, is in fact observing Elphaba while pretending to read a newspaper. The total amount of time Elphaba has spent making eye contact with Erin is a function of the number of times that Erin looks up from the newspaper. Let $g(n)$ be the total amount of time, in seconds, that Erin and Elphaba have spent making eye contact if Erin has looked up from her newspaper n times.

- a. [2 points] Using a complete sentence, give a practical interpretation of the expression $f^{-1}(3) = 10$. Be sure to include units.

Solution: After Elphaba has made eye contact with Erin for a total of 10 seconds, her anxiety is at 3 anxious units.

Alternative: Elphaba's anxiety is at 3 anxious units when she has made eye contact with Erin for a total of 10 seconds.

- b. [3 points] Below is the first part of a sentence that will give a practical interpretation of the equation

$$f'(25) = 2.$$

Complete the sentence so that the practical interpretation can be understood by someone who knows no calculus. Be sure to include units in your answer.

If Elphaba has already made eye contact with Erin for a total of 25 seconds and she makes eye contact for an additional 0.3 seconds, then

Solution: If Elphaba has already made eye contact with Erin for a total of 25 seconds and she makes eye contact for an additional 0.3 seconds, then Elphaba's anxiety will increase by approximately 0.6 anxious units.

(Note: $0.6 = 2 \times 0.3$)

- c. [2 points] Given that $(f^{-1})'(99) = 7$ and $f(62) = 99$, approximate the total length of time Elphaba has to spend making eye contact with Erin before she will panic and run.

Solution: The expression $(f^{-1})'(99) = 7$ means that once Elphaba's anxiety is at 99 anxious units, it takes approximately 7 seconds for her anxiety to reach 100 anxious units. The expression $f(62) = 99$ means that it takes 62 seconds for Elphaba's anxiety to reach 99 anxious units. Therefore putting these together, it will take approximately $62 + 7 = 69$ seconds for Elphaba's anxiety to reach 100 anxious units, which is when she will panic and run.

- d. [3 points] Which of the following sentences gives a correct interpretation of the quantity $g^{-1}(f^{-1}(50))$? Circle the ONE best answer.

i. When Erin has looked up from her newspaper 50 times, Elphaba's anxiety is at $g^{-1}(f^{-1}(50))$ anxious units.

ii. When Erin has looked up from her newspaper 50 times, Erin and Elphaba have spent $g^{-1}(f^{-1}(50))$ seconds making eye contact.

iii. If Erin has looked up from her newspaper $g^{-1}(f^{-1}(50))$ times then Elphaba's anxiety is 50 anxious units.

iv. If Erin and Elphaba have made eye contact for a total of 50 seconds then Erin has looked up from her newspaper $g^{-1}(f^{-1}(50))$ times.

v. When Erin and Elphaba have made eye contact for a total of 50 seconds then Elphaba's anxiety is at 50 anxious units.