4. [10 points] For each of the following, give a *formula* for a single function satisfying all of the listed properties. If there is no function satisfying all the properties, circle NO SUCH FUNCTION EXISTS.

Note: If "NO SUCH FUNCTION EXISTS" is circled, then any formula you have written will not be graded.

a. [3 points] A *polynomial* $p(t)$ with the following three properties:

- The degree of $p(t)$ is three.
- $p(t) \to -\infty$ as $t \to \infty$.
- $p(0) = -4$.

Solution: Note that the second property implies that the leading coefficient of the polynomial is negative, and the third property implies that, when written in standard form, the constant term of $p(t)$ is -4. So one example is $p(t) = -t^3 - 4$.

Answer: $p(t) =$ $-t^3 - 4$ OR Circle: NO SUCH FUNCTION EXISTS

b. [3 points] An *exponential function* $q(v)$ with the following three properties:

- $q(1) = 3$.
- $\lim_{v\to 0} q(v) = 12$.
- $\lim_{v\to\infty} q(v) = 0$.

Solution: Since exponential functions are continuous, the second property implies that $q(0) = 12$. So $q(v)$ is exponential with initial value 12 and decay factor equal to $\frac{q(1)}{q(0)} = \frac{3}{12} = \frac{1}{4}$. Therefore q must be the function given by $q(v) = 12\left(\frac{1}{4}\right)^v$.

Answer: $q(v) =$ $12\left(\frac{1}{4}\right)^v$ OR Circle: NO SUCH FUNCTION EXISTS

c. [4 points] A *rational* function $r(x)$ with the following three properties:

- The line $x = 2$ is a vertical asymptote of the graph of $y = r(x)$.
- The line $y = -3$ is a horizontal asymptote of the graph of $y = r(x)$.
- $r(5) = 0$.

Solution: These properties imply that $r(x)$ can be written as a quotient of polynomials $\frac{p(x)}{q(x)}$ such that $(x-2)$ is a factor of $q(x)$, the ratio of the leading term of $p(x)$ to that of $q(x)$ is -3, and $(x-5)$ is a factor of $p(x)$. There are many possibilities, but one example is $r(x) = \frac{-3(x-5)}{x-2}$.

Answer: $r(x) =$ $\dfrac{-3(x-5)}{x-2}$ OR Circle: NO SUCH FUNCTION EXISTS