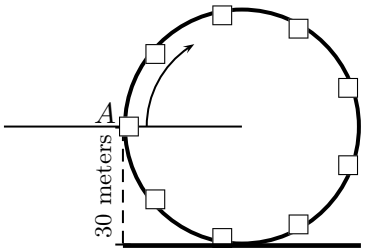
6. [10 points] Erin is in pursuit of squirrel and suspected criminal, Elphaba. Suddenly there is a cliff ahead. Elphaba, who it turns out is a flying squirrel, jumps straight off and glides safely down to the ground. Searching for an alternative, Erin finds a ferris wheel that will take her to the ground beneath the cliff.

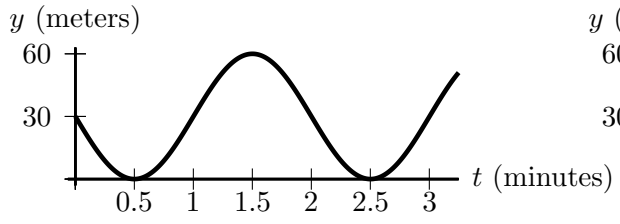
The ferris wheel has radius 30 meters and is rotating (clockwise in the diagram shown) at a constant rate of one half radian per minute. Let $H(t)$ be Erin's height above the ground beneath the cliff (in meters) t minutes after she gets on the ferris wheel. A diagram of the situation is shown to the right. Note that Erin gets on the ferris wheel at position A, and $H(0) = 30$.



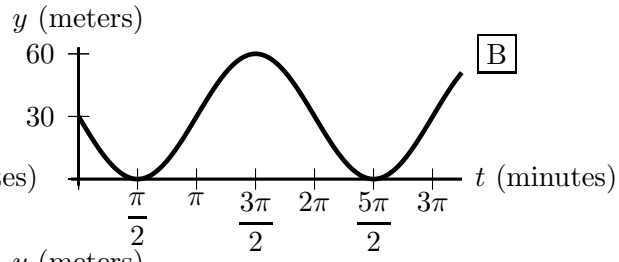
a. [2 points] Which of the following graphs is a graph of $y = H(t)$?
Write the letter (A-D) of the ONE *best choice.* **Answer:** **C**

Solution: Erin is 30 meters above the ground at time $t = 0$, and her height above the ground increases at first. Since the ferris wheel is rotating at a constant rate of 1/2 radian per minute, it will take 4π minutes for it to rotate 2π radians. So the period of $H(t)$ is 4π. The graph shown as C below is the best choice.

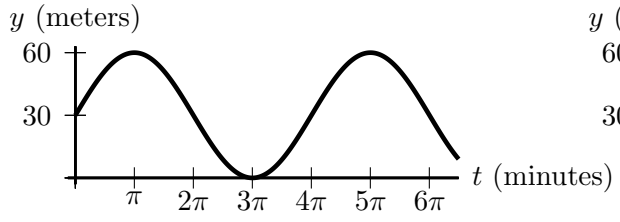
A



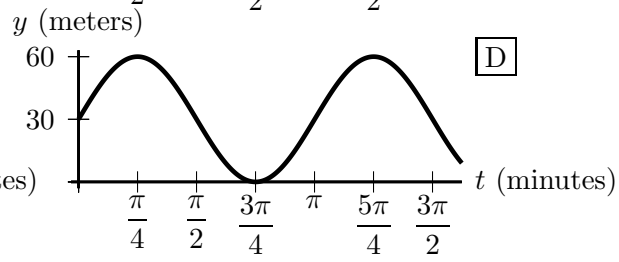
B



C



D



b. [4 points] Write a formula for the sinusoidal function $H(t)$.

Solution: $H(t)$ is a sinusoidal function with amplitude 30, midline $y = 30$, and period 4π. The value is on the midline and increasing when $t = 0$. There are many possible formulas. One is $H(t) = 30\sin(\frac{t}{2}) + 30$.

Answer: $H(t) =$ $30\sin\left(\frac{t}{2}\right) + 30$

c. [4 points] Erin figures that if she jumps off when she is no more than b meters above the ground, where b is a constant between 0 and 30, then she will be fine. Erin would like to jump off before she has to go around the ferris wheel again. What is the latest time she can jump off without going around a full revolution? Remember to show your work clearly. Your answer may involve the constant b.

Solution: We are looking for a particular solution to the equation $30\sin(t/2) + 30 = b$. Using inverse trig to solve this, we find that one solution is $t = 2\sin^{-1}(\frac{b-30}{30})$. Now we need to think about what point on the graph this actually gives us. In this case, since $-1 < \frac{b-30}{30} < 0$ this solution is the first solution to the left of the vertical axis in the graph above. We need to add the period 4π to get the time we are looking for. So to avoid going around a full revolution, the latest time Erin can jump off is $2\sin^{-1}\left(\frac{b-30}{30}\right) + 4\pi$ minutes after she got on the ferris wheel.

Answer: $t = 2\sin^{-1}\left(\dfrac{b-30}{30}\right) + 4\pi$