7. [10 points] Sebastian has chartered a helicopter which is rising straight up in the air, but he is scared of heights. Let $A(w)$ be Sebastian's fear (in "scared units") when he is w km above the ground. For $0 < w \le 2$, a formula for $A(w)$ is given by

$$A(w) = \frac{w^2+2}{w^w+1}.$$

a. [5 points] Use the limit definition of the derivative to write an explicit expression for the instantaneous rate of change of Sebastian's fear, in scared units per km, when he is 1.5 km above the ground. *Your answer should not involve the letter A. Do not attempt to evaluate or simplify the limit.*

Solution: The instantaneous rate of change of Sebastian's fear, in scared units per km, is given by the derivative

$$A'(1.5) = \lim_{h\to 0} \frac{A(1.5+h) - A(1.5)}{h}.$$

Answer: $A'(1.5) = \boxed{\lim_{h\to 0} \dfrac{\dfrac{(1.5+h)^2+2}{(1.5+h)^{(1.5+h)}+1} - \dfrac{1.5^2+2}{1.5^{1.5}+1}}{h}}$

b. [5 points] When he has reached a height of 2 km above the ground Sebastian gets control of his fear and his fear starts decreasing at a constant rate of 0.8 scared units per km. Write a formula for a piecewise-defined continuous function $A(w)$ giving Sebastian's fear, in scared units, for $0 < w < 3$.

Solution: We were given that $A(w) = \frac{w^2+2}{w^w+1}$ for $0 < w \le 2$. So it remains to find a formula for $A(w)$ that is valid for $2 < w < 3$. Since Sebastian's fear is decreasing at a constant rate for $w > 2$, $A(w)$ is linear for $2 < w < 3$, and the slope of this linear piece is -0.8. In order for $A(w)$ to be continuous, this linear piece must pass through the point $(2, A(2))$ which is $(2, 6/5)$. Using point slope form, this gives the formula $1.2 - 0.8(w-2) = 2.8 - 0.8w$ for the linear piece.

Answer: $A(w) = \begin{cases} \dfrac{w^2+2}{w^w+1} & \text{if } 0 < w \le 2 \\ 2.8 - 0.8w & \text{if } 2 < w < 3. \end{cases}$