8. [10 points] *Throughout this page, give all answers in exact form. Do not use decimal approximations. For example,* $x = \frac{1}{3}$ *is an exact solution to* $3x = 1$*, but* $x = 0.3333333333$ *is not.*

Sebastian has rented a helicopter to catch up to his friend Erin who is currently chasing a suspected criminal named Elphaba. When Sebastian first sees the pair they are 180 meters apart. After 3 minutes, Erin has moved 60 meters closer to Elphaba. (In other words, the distance between them has decreased by 60 meters.) Let $D(t)$ be the distance between Elphaba and Erin, in meters, t minutes after Sebastian begins watching them.

a. [2 points] Sebastian initially assumes that $D(t)$ is a linear function. Find a formula for $D(t)$ under this assumption, valid for as long as it takes for Erin to catch Elphaba.

Solution: If $D(t)$ is linear, then $D(t) = b + mt$ where b is the initial value and m is the constant average rate of change. We immediately know $b = 180$ since this is the initial value. We then know the function decreases by 60 m in 3 minutes so that's an average rate of decrease of 20m/min. Therefore $D(t) = 180 - 20t$.

Answer: $D(t) =$ $180 - 20t$

b. [1 point] After Sebastian has been watching for 6 minutes, the distance between Erin and Elphaba is 80 meters. Briefly explain why this contradicts Sebastian's initial assumption.

Solution: One way to see is that the point $(6, 80)$ doesn't satisfy the formula we found in a. Another explanation would be that the average rate of change between $(0, 180)$ and $(3, 120)$ is different from the average rate of change between $(3, 120)$ and $(6, 80)$.

c. [4 points] Sebastian then determines that $D(t)$ must in fact be an exponential function. Write a new formula for $D(t)$ given this new information (including the data from part (b)). *Remember to show your work carefully and use exact form.*

Solution: We now know $D(t) = bc^t$ where b is again the initial value, so $D(t) = 180c^t$. We can solve for c by using the fact that $D(3) = 120$. Then $120 = 180c^3$ so $c = (\frac{120}{180})^{1/3} = \left(\frac{2}{3}\right)^{1/3}$.

Answer: $D(t) =$ $180((\frac{120}{180})^{\frac{1}{3}})^t = 180\left(\frac{2}{3}\right)^{t/3}$

d. [3 points] Erin can catch Elphaba when she is within one meter of her (since Erin can jump and tackle Elphaba at this distance). Use algebra and your formula from part (c) to find how long it takes for the distance between Erin and Elphaba to decrease to 1 meter.

Solution: We need to solve for t in the equation $D(t) = 1$. Using our formula from part (c), we have $180((\frac{2}{3})^{\frac{1}{3}})^t = 1$. Solving, we find $((\frac{2}{3})^{\frac{1}{3}})^t = \frac{1}{180}$ so $t\ln((\frac{2}{3})^{\frac{1}{3}}) = \ln(\frac{1}{180})$. Finally $t = \frac{\ln(\frac{1}{180})}{\ln((\frac{2}{3})^{\frac{1}{3}})} = \frac{-3\ln(180)}{\ln(2)-\ln(3)} = \frac{3\ln(180)}{\ln(1.5)}$. So it takes $\frac{3\ln(180)}{\ln(1.5)}$ (about 38.4) minutes for Erin to get within 1 meter of Elphaba.

Answer: $\dfrac{\ln\left(\frac{1}{180}\right)}{\ln\left(\left(\frac{2}{3}\right)^{\frac{1}{3}}\right)} = \dfrac{3\ln(180)}{\ln(1.5)}$