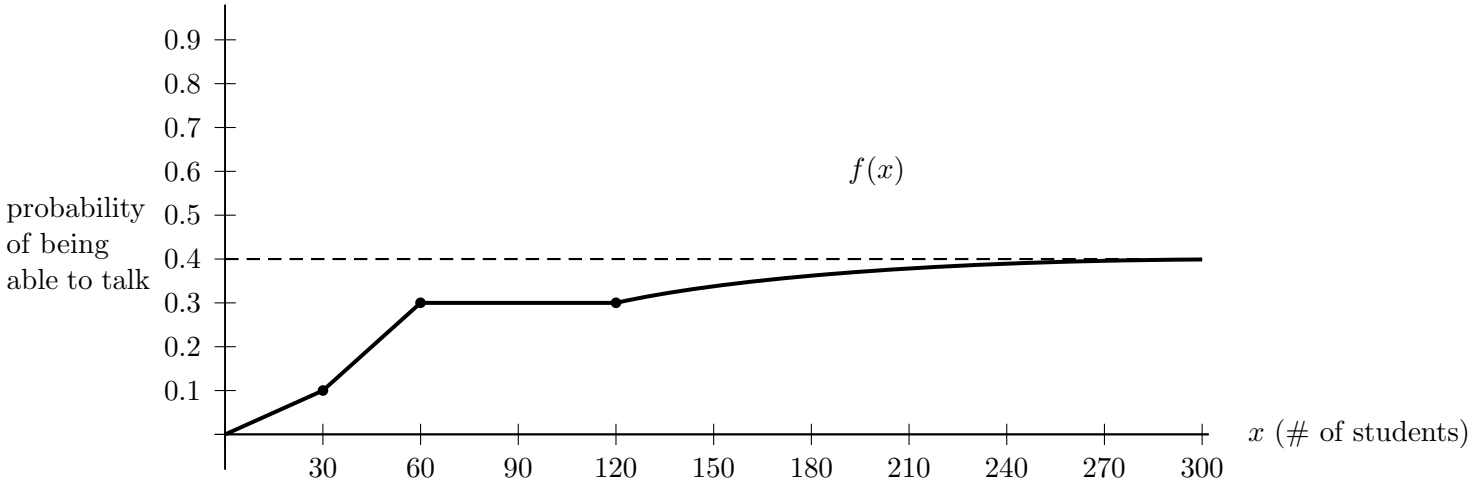
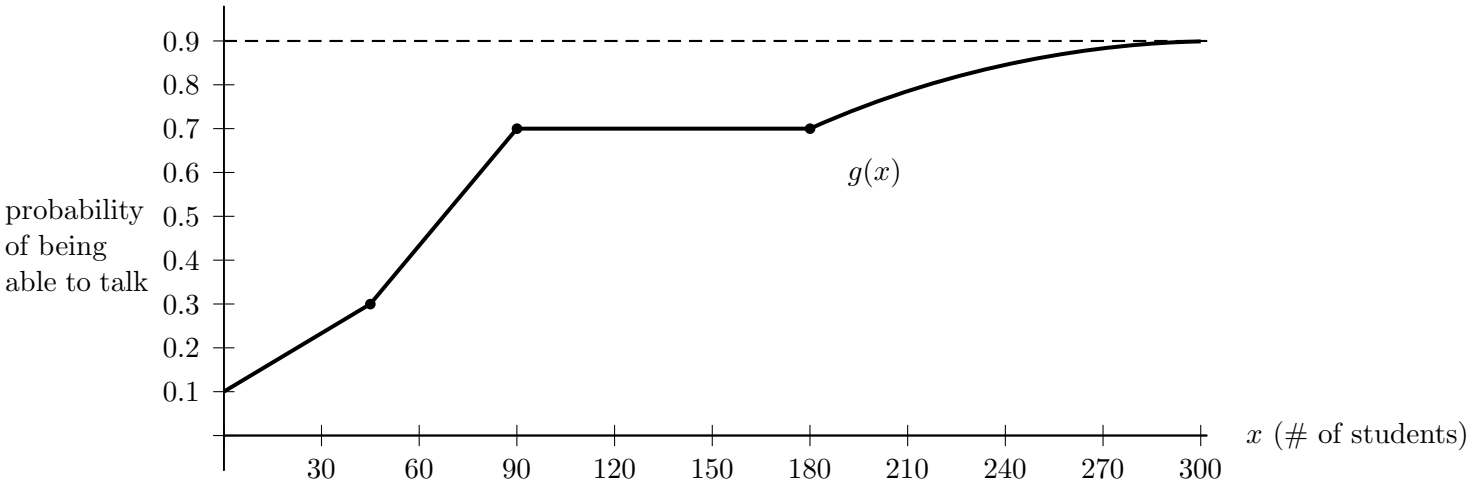
9. [12 points] Lauren has just been approached by a talking kangaroo named Skipper. Lauren is alarmed by Skipper's ability to talk so she asks him if all kangaroos can talk. Skipper tells her that a kangaroo's probability of being able to talk depends on how much attention they receive from University of Sydney students. He also explains that this relationship has changed recently. Let $f(x)$ be the probability that a kangaroo born prior to 5 weeks ago is able to talk if x students have paid attention to that kangaroo in their life. Below is a graph of the function $f(x)$.



a. [4 points] Let $g(x)$ be the probability that a kangaroo born within the last 5 weeks is able to talk if x students have paid attention to that kangaroo in their life. A graph of $g(x)$ is given below.



It turns out that for $x \geq 0$, $g(x)$ can be expressed as a transformation of $f(x)$. Write a formula for $g(x)$ as a transformation of $f(x)$.

Solution: To obtain the graph of $g(x)$ from that of $f(x)$, we first vertically stretch by a factor of 2, then shift the resulting graph up by 0.1 units, and finally stretch it horizontally by a factor of 3/2.

Answer: $g(x) = 2f\left(\frac{2}{3}x\right) + 0.1$

Problem continues on the next page.

This is a continuation of the problem from the previous page.

b. [3 points] Let $h(y)$ be the number of students that pay attention to a kangaroo over the duration of the kangaroo's life if the kangaroo has y docile units. (Note that we measure docileness in terms of docile units.) Give a practical interpretation of $g(h(5))$.

Solution: $g(h(5))$ is the probability that a kangaroo born in the last 5 weeks with 5 docile units will be able to talk.

c. [3 points] The number of students who pay attention to a kangaroo with y docile units is proportional to y^2. Find a formula for $h(y)$ if a total of 160 students pay attention to a kangaroo with 4 docile units during its life.

Solution: The first sentence tells us that $h(y) = ky^2$ for some constant k. Using the fact that $h(4) = 160$, we find $160 = k(4^2)$, so $160 = 16k$ and $k = 10$.

Answer: $h(y) =$ $10y^2$

d. [2 points] Calculate $f(h(3))$.

Solution: Using our formula from part c, we have $h(3) = 10(3^2) = 90$ so $f(h(3)) = f(90) = 0.3$.

Answer: $f(h(3)) =$ 0.3