3. [4 points] Let $h(x) = (x+3)e^{2x-2}$. Then the derivative of h is given by the formula $h'(x) = (2x+7)e^{2x-2}$. Find an equation for the tangent line to the graph of $y = h(x)$ at $x = 1$.

Solution: Because

$$h(1) = (1+3)e^{2(1)-2} = 4$$
$$h'(1) = (2(1)+7)e^{2(1)-2} = 9$$

the tangent line has slope 9 and goes through the point $(1, 4)$, so to get the formula for the tangent line:

$$y - 4 = 9(x-1)$$
$$y = 9x - 5$$

Answer: $y =$ $9x - 5$

4. [10 points] Consider the function g defined by $g(x) = \begin{cases} \dfrac{1}{e^x - 1} & \text{if } x < \frac{1}{2} \\ \cos(x^x) & \text{if } \frac{1}{2} \le x < 5 \\ \dfrac{x^2}{(x-1)(6-x)} & \text{if } x \ge 5. \end{cases}$

a. [5 points] Use the limit definition of the derivative to write an explicit expression for $g'(3)$. *Your answer should not involve the letter g. Do not attempt to evaluate or simplify the limit.* Please write your final answer in the answer box provided below.

Solution: $g'(3) = \lim\limits_{h\to 0} \dfrac{\cos\left((3+h)^{3+h}\right) - \cos\left(3^3\right)}{h}$

Answer: $g'(3) = \boxed{\lim\limits_{h\to 0} \dfrac{\cos\left((3+h)^{3+h}\right) - \cos\left(3^3\right)}{h}}$

b. [3 points] Find all vertical asymptotes of the graph of $g(x)$. If there are none, write NONE.

Solution: Note that $x = 1$ is *not* a vertical asymptote because the third piece of the formula for $g(x)$ is only valid for $x \ge 5$. The vertical asymptotes are$x = 0$ and $x = 6$.

Answer: $x = 0$**,** $x = 6$

c. [2 points] Determine $\lim\limits_{x\to\infty} g(x)$. If the limit does not exist, write DNE.

Solution: $\lim\limits_{x\to\infty} g(x) = \lim\limits_{x\to\infty} \dfrac{x^2}{(x-1)(6-x)} = \lim\limits_{x\to\infty} \dfrac{x^2}{-x^2+7x-6} = -1.$

Answer: -1