

5. [10 points] Vikram takes a non-stop train ride from Chennai straight to New Delhi. Let $g(t)$ be the distance (in km) of Vikram's train from Chennai t hours after his ride begins. Assume that the function g is increasing and invertible, and that g and g^{-1} are differentiable. Several values for $g(t)$ are shown in the table below.

t	0	2	5	6.5	10	11	16	28
$g(t)$	0	132	346	448	692	742	1152	2180

- a. [3 points] Estimate the instantaneous velocity of Vikram's train 6 hours after his ride begins. *Show your work and include units.*

Solution: We estimate using average velocity based on nearby measurements:

$$\frac{448 - 346}{6.5 - 5} = 68$$

So we estimate the instantaneous velocity of Vikram's train 6 hours after his ride begins to be about 68 km/h.

Answer: 68 km/h

- b. [5 points] Suppose $(g^{-1})'(700) = C$, where C is some constant.
 (i) Using the data in the table above, find the best possible estimate of C .
Show your work.

Solution: We estimate the derivative based on nearby measurements:

$$\frac{11 - 10}{742 - 692} = 0.02 \text{ h/km}$$

So we estimate $C \approx 0.02$. 700 has units km, and C has units h/km.

Answer: 0.02

- (ii) In interpreting the equation $(g^{-1})'(700) = C$, what are the units on 700 and C ?

Answer: Units on 700 are km

Answer: Units on C are h/km

- c. [2 points] Let $R(t)$ be the total rainfall (in cm) in New Delhi during the first t hours of Vikram's train ride. Express the following statement with a single mathematical equation: "Over the first 900 km of Vikram's train ride, it rained 3.6 cm in New Delhi."

Answer: $R(g^{-1}(900)) = 3.6$