

10. [4 points] Find all real numbers  $B$  and positive integers  $k$  such that the rational function

$$H(x) = \frac{9 + x^k}{16 - Bx^3}$$

satisfies the following two conditions:

- $H(x)$  has a vertical asymptote at  $x = 2$
- $\lim_{x \rightarrow \infty} H(x)$  exists.

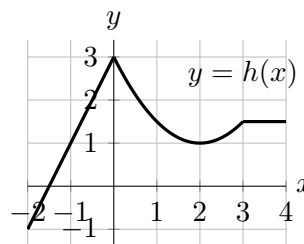
If no such values exist, write NONE.

**Justification:** In order for  $\lim_{x \rightarrow \infty} H(x)$  to exist, the degree of the polynomial in the numerator has to be smaller or equal to 3 (the degree of the polynomial in the denominator). In order for  $x = 2$  to be a vertical asymptote, you need the denominator to be zero. Hence  $16 - B(2^3) = 16 - 8B = 0$  which requires  $B = 2$ . In this case  $H(x) = \frac{9 + x^k}{16 - 2x^3}$  with  $k = 1, 2$  or  $3$ . Since  $9 + 2^k \neq 0$ , then  $H(x)$  has a vertical asymptote at  $x = 2$  when  $B = 2$ .

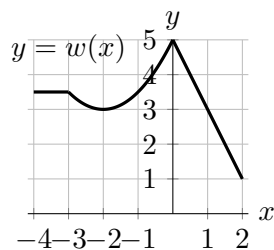
**Answer:**  $B =$  2 **Answer:**  $k =$  1, 2, or 3

11. [4 points] A part of the graph of a function  $h(x)$  is given below.

In each of the following parts, the corresponding portion of the graph of a function obtained from  $h$  by one or more transformations is shown, together with a list of possible formulas for that function. In each case choose the one correct formula for the function shown. *Note that the graphs are not all drawn at the same scale.*

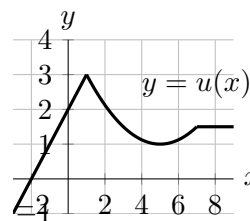


a. [2 points]



- |                |                  |
|----------------|------------------|
| A. $h(-x) - 2$ | F. $-h(x - 2)$   |
| B. $-h(x) - 2$ | G. $-h(-x + 2)$  |
| C. $-h(x) + 2$ | H. $-h(-x - 2)$  |
| D. $h(-x) + 2$ | I. $h(-x + 2)$   |
| E. $-h(x + 2)$ | J. $h(-x - 2)$   |
|                | K. NONE OF THESE |

b. [2 points]



- |                      |                      |
|----------------------|----------------------|
| A. $h(0.5x + 1)$     | F. $h(2x - 1)$       |
| B. $h(0.5x - 1)$     | G. $h(2x + 1)$       |
| C. $h(0.5(x - 1))$   | H. $h(2(x - 1))$     |
| D. $h(0.5(x + 1))$   | I. $h(2(x + 1))$     |
| E. $h(0.5(x - 0.5))$ | J. $h(0.5(x + 0.5))$ |
|                      | K. NONE OF THESE     |